Deconstructing Space-Frequency Correlated Ultrawideband MIMO Channels

(*Invited Paper*)

Xuemin Hong[1], Cheng-Xiang Wang[1], John Thompson[2], Ben Allen[3], and Wasim Q. Malik[4]
[1]Joint Research Institute for Signal and Image Processing, Heriot-Watt University, Edinburgh, EH14 4AS, UK.
[2]Joint Research Institute for Signal and Image Processing, University of Edinburgh, Edinburgh, EH9 3JL, UK.
[3]Department of Engineering Science, University of Oxford, Oxford, OX1 3PJ, UK.
[4]Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.
Email: xh12@hw.ac.uk, cheng-xiang.wang@hw.ac.uk, john.thompson@ed.ac.uk, ben.allen@eng.ox.ac.uk, wqm@mit.edu

***Abstract*—Conventional correlation models fail to accurately represent the correlation properties of ultrawideband (UWB) multiple-input multiple-output (MIMO) channels. In our previous work, a framework for constructing correlated UWB MIMO channel models was proposed, where spatial correlation was introduced into both the multipath amplitude and time-of-arrival (ToA) in the channel impulse response. Based on this framework, in this paper we first present a simplified UWB MIMO channel model that represents a reasonable compromise between analytical tractability and model accuracy. We show that this model yields a structured space-frequency (SF) channel covariance matrix, making it suitable for theoretical system performance analysis. To illustrate the merit of this model, we apply it to evaluate the maximum diversity order in SF coded UWB MIMO systems. In contrast to previous work based on conventional wideband MIMO correlated channel models, we show that the maximum diversity order of a UWB MIMO system is not limited by the sum of the ranks of multipath amplitude correlation matrices.**

I. Introduction

Ultrawideband (UWB) is a dynamic spectrum access (DSA) technique that aims to improve the radio spectrum utilization [1]. Based on an underlay DSA approach [1], a UWB system seeks to coexist with the incumbents by spreading the transmit power over a very wide bandwidth. It thus holds great promise for enabling unlicensed short-range high-speed wireless access [2]. A UWB MIMO system deploys antenna arrays at both ends of the communication link and can be used to increase the data rates and/or extend the coverage [3]. Unfortunately, the performance of UWB MIMO systems degrades in the presence of channel correlation, which is often encountered in practical situations due to insufficient antenna spacing or sparse scattering. Consequently, an accurate and analytically tractable channel correlation model is critical for the design and performance analysis of realistic UWB MIMO systems.

Conventional correlation models for wideband MIMO channels with a tapped-delay-line structure assume that multipath components (MPCs) arrive at different antenna elements with varying (and possibly correlated) complex amplitudes but identical ToAs [4]–[6]. Such models become inappropriate in the UWB regime due to the high delay resolution, which results in distinguishable ToA differences even between closely-spaced antenna elements. In our previous work [7], we proposed a UWB MIMO channel modeling framework that considers both the amplitude and ToA correlations. The resulting channel model was shown to better fit the measurement results. Based on this framework, in this paper we aim to investigate the inherent structure of the UWB MIMO channels, understand the difference between UWB and wideband MIMO channels, and discuss the implications of the channel structure from the signal processing and information theoretic perspectives.

II. UWB MIMO Channel Model

We consider an $M_t \times M_r$ UWB MIMO channel, with M_t and M_r denoting the number of transmit and receive antenna elements, respectively. The channel impulse response from the pth ($p = 1, 2, \ldots, M_t$) transmit antenna to the qth ($q = 1, 2, \ldots, M_r$) receive antenna is given by

$$h^{p,q}(t) = \sum_{l=0}^{L-1} a_l^{p,q} \delta(t - \tau_l - \epsilon_l^{p,q}) \quad (1)$$

where l ($l = 0, 1, 2, \ldots, L-1$) is the MPC index, L is the number of MPCs, $a_l^{p,q}$ is the amplitude coefficient of the lth MPC, τ_l is the reference ToA of the lth MPC, and $\epsilon_l^{p,q}$ is the ToA difference with respect to τ_l. Following the IEEE 802.13a UWB channel model, the amplitude coefficient $a_l^{p,q}$ is taken to be real-valued random variable. Note that the above channel model in (1) differs from conventional tapped-delay-line wideband MIMO channel models, in which $a_l^{p,q}$ has complex value and $\epsilon_l^{p,q}$ is assumed to be 0.

Given an impinging MPC, the amplitude and ToA values perceived by different antenna elements are in general varied but correlated. The amplitude variation is caused by the well-known small scale fading phenomenon [2], while the ToA variation is simply caused by the propagation delays among antenna elements. In [7], it was proposed to study the spatial correlation properties of UWB MIMO channels in terms of both amplitude correlation and ToA correlation.

On one hand, the amplitude correlation of the lth MPC is characterized by a $M_tM_r \times M_tM_r$ covariance matrix $\mathbf{R}_A(l)$ given by

$$\mathbf{R}_A(l) = E\left\{\mathbf{a}_l\mathbf{a}_l^H\right\} \quad (2)$$

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where $E\{\cdot\}$ denotes statistical expectation, $[\cdot]^H$ stands for Hermitian transpose of a matrix/vector. In (2), $\mathbf{a}_l$ is a $M_tM_r \times 1$ vector formatted as

$$\mathbf{a}_l = \left[a_l^{1,1}, \cdots, a_l^{1,M_r}, a_l^{2,1}, \cdots, a_l^{M_t,1}, \cdots, a_l^{M_t,M_r}\right]^T \quad (3)$$

where $[\cdot]^T$ stands for matrix transpose. An arbitrary entry of $\mathbf{R}_A(l)$ is denoted as

$$\chi_l^{p_1q_1,p_2q_2} = E\left\{a_l^{p_1,q_1}(a_l^{p_2,q_2})^*\right\} \quad (4)$$

where $p_1, p_2 = 1, 2, \ldots, M_t$, $q_1, q_2 = 1, 2, \ldots, M_r$, and $(\cdot)^*$ denotes complex conjugate.

On the other hand, the ToA correlation can be captured by the ToA difference $\epsilon_l^{p,q}$. When both the transmitter and receiver deploy uniform linear arrays (ULAs), $\epsilon_l^{p,q}$ is given by [7]

$$\epsilon_l^{p,q} = \left[d_p \sin(\theta_{l,AoA}) + d_q \sin(\theta_{l,AoD})\right]/c \quad (5)$$

where c is the speed of light, d_p and d_q are the corresponding transmit and receive antenna spacings, respectively, and $\theta_{l,AoA}$ and $\theta_{l,AoD}$ are the angle-of-arrival (AoA) and angle-of-departure (AoD) of the lth MPC, respectively. Unlike [7], where $\theta_{l,AoA}$ and $\theta_{l,AoD}$ are treated as correlated and obtained from complex hierarchical angular models, in this paper we simply assume that $\theta_{l,AoA}$ and $\theta_{l,AoD}$ of all the MPCs follow independent and identical uniform distributions from 0 to 2π. Such a simplification can significantly improve the analytical tractability of the model and is reasonable when we consider the averaging effect of a large number of channel realizations.

III. Deconstructing the SF Channel Covariance Matrix

Given the channel impulse response in (1), the corresponding channel frequency response $H^{p,q}(f)$ can be obtained by applying the Fourier transform. In the context of orthogonal frequency-division multiplexing (OFDM) based UWB MIMO systems [6], we are interested in the discretized channel frequency response $H^{p,q}(k)$ given by

$$H^{p,q}(k) = \sum_{l=0}^{L-1} a_{k,l}^{p,q} \exp\left[-j2\pi k\Delta f(\tau_l + \epsilon_l^{p,q})\right] \quad (6)$$

where Δf denotes the subcarrier frequency spacing, $k = 1, 2, \ldots, N$ denotes the subcarrier index, and N denotes the total number of subcarriers. The channel vector $\mathbf{H}$ of size $M_tM_rN \times 1$ is formatted as

$$\mathbf{H} = \left[\mathbf{H}^{1,1}, \cdots, \mathbf{H}^{1,M_r}, \cdots, \mathbf{H}^{p,q}, \cdots, \mathbf{H}^{M_t,1}, \cdots, \mathbf{H}^{M_t,M_r}\right]^T \quad (7)$$

where $\mathbf{H}^{p,q}$ is a $N \times 1$ vector given by

$$\mathbf{H}^{p,q} = \left[H^{p,q}(1), H^{p,q}(2), \cdots, H^{p,q}(N)\right]. \quad (8)$$

We then have $\mathbf{R} = E\left\{\mathbf{H}\mathbf{H}^H\right\}$ as the channel covariance matrix. The matrix $\mathbf{R}$ is of size $M_tM_rN \times M_tM_rN$ and is also referred to as the SF channel covariance matrix in this paper.

A. Entries of $\mathbf{R}$

Consider an arbitrary entry of $\mathbf{R}$ given by

$$\rho_{k_1,k_2}^{p_1q_1,p_2q_2} = E\left\{H^{p_1,q_1}(k_1)\left[H^{p_2,q_2}(k_2)\right]^*\right\} \quad (9)$$

where $k_1, k_2 = 1, 2, \ldots, N$. Substituting (6) into (9), we have

$$\rho_{k_1,k_2}^{p_1q_1,p_2q_2} = \alpha_{k_1,k_2}^{p_1q_1,p_2q_2}\beta_{k_1,k_2}^{p_1q_1,p_2q_2} \quad (10)$$

where

$$\alpha_{k_1,k_2}^{p_1q_1,p_2q_2} = \sum_{l=0}^{L-1} \chi_l^{p_1q_1,p_2q_2} \exp\left[-j2\pi\Delta f(k_1 - k_2)\tau_l\right] \quad (11)$$

is a coefficient related to the amplitude correlation and

$$\beta_{k_1,k_2}^{p_1q_1,p_2q_2} = E\left\{\exp\left[-j2\pi\Delta f(k_1\epsilon_l^{p_1,q_1} - k_2\epsilon_l^{p_2,q_2})\right]\right\} \quad (12)$$

is a coefficient related to the ToA correlation. In conventional wideband MIMO channels, the differences among multipath ToAs are not taken into account, i.e., $\epsilon_l^{p_1,q_1} = \epsilon_l^{p_2,q_2} = 0$. It follows that $\beta_{k_1,k_2}^{p_1q_1,p_2q_2} = 1$ and therefore $\rho_{k_1,k_2}^{p_1q_1,p_2q_2} = \alpha_{k_1,k_2}^{p_1q_1,p_2q_2}$.

B. Decomposition of $\mathbf{R}$

In this section, we will show that the channel covariance matrix $\mathbf{R}$ can be written in a compact form using some matrix operations. This property gives the proposed channel model the advantage of facilitating theoretical analysis of UWB MIMO systems. According to (10), we can write

$$\mathbf{R} = \mathbf{A} \circ \mathbf{B} \quad (13)$$

where $\circ$ denotes the Hadamard (entry-wise) product, $\mathbf{A}$ is a $M_tM_rN \times M_tM_rN$ matrix whose entries are given by $\alpha_{k_1,k_2}^{p_1q_1,p_2q_2}$, and $\mathbf{B}$ is also a $M_tM_rN \times M_tM_rN$ matrix whose entries are given by $\beta_{k_1,k_2}^{p_1q_1,p_2q_2}$. From (11), it can be shown that $\mathbf{A}$ can be written as

$$\mathbf{A} = (\mathbf{I}_{M_tM_r} \otimes \mathbf{W})\,\mathbf{\Phi}\left(\mathbf{I}_{M_tM_r} \otimes \mathbf{W}^H\right) \quad (14)$$

where $\otimes$ denotes the Kronecker product, $\mathbf{I}_{M_tM_r}$ is the $M_tM_r \times M_tM_r$ identity matrix, and $\mathbf{W}$ is an $N \times L$ Fourier matrix defined as

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(N-1)\tau_0} & w^{(N-1)\tau_1} & \cdots & w^{(N-1)\tau_{L-1}} \end{bmatrix} \quad (15)$$

where $w = \exp(-j2\pi\Delta f)$. In (14), $\mathbf{\Phi}$ is a $M_tM_rL \times M_tM_rL$ square matrix defined as

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{V}^{1111} & \mathbf{V}^{1112} & \cdots & \mathbf{V}^{11M_tM_r} \\ \mathbf{V}^{1211} & \mathbf{V}^{1212} & \cdots & \mathbf{V}^{12M_tM_r} \\ \vdots & \vdots & \mathbf{V}^{p_1q_1p_2q_2} & \vdots \\ \mathbf{V}^{M_tM_r11} & \mathbf{V}^{M_tM_r12} & \cdots & \mathbf{V}^{M_tM_rM_tM_r} \end{bmatrix} \quad (16)$$

where $\mathbf{V}^{p_1q_1p_2q_2}$ is a $L \times L$ diagonal matrix given by

$$\mathbf{V}^{p_1q_1p_2q_2} = \mathrm{diag}\left\{\chi_0^{p_1q_1,p_2q_2}, \chi_1^{p_1q_1,p_2q_2}, \cdots, \chi_{L-1}^{p_1q_1,p_2q_2}\right\}. \quad (17)$$

On the other hand, substituting (5) into (12), after some mathematical manipulation, matrix $\mathbf{B}$ can be written as

$$\mathbf{B} = J_0\left(\frac{2\pi\Delta f}{c}(\mathbf{P}-\mathbf{P}^T)\right) \circ J_0\left(\frac{2\pi\Delta f}{c}(\mathbf{Q}-\mathbf{Q}^T)\right) \quad (18)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, $\mathbf{P}$ and $\mathbf{Q}$ are both $M_tM_rN \times M_tM_rN$ matrices given by

$$\mathbf{P} = \mathbf{d}_T \otimes \mathbf{1}_{M_tM_rN\times M_r} \otimes \mathbf{d}_N \quad (19)$$
$$\mathbf{Q} = \mathbf{1}_{M_tM_rN\times M_r} \otimes \mathbf{d}_R \otimes \mathbf{d}_N \quad (20)$$

respectively. Here, $\mathbf{d}_T = d_p[0, 1, \cdots, M_t-1]$ and $\mathbf{d}_R = d_q[0, 1, \cdots, M_r-1]$ denote the $1 \times M_t$ transmit antenna spacing vector and $1 \times M_r$ receive antenna spacing vector, respectively, $\mathbf{d}_N = [1, 2, \cdots, N]$ is the $1\times N$ subcarrier index vector, and $\mathbf{1}_{M_tM_rN\times M_r}$ is a $M_tM_rN \times M_r$ matrix with unit entries.

We can see that the SF covariance matrix $\mathbf{R}$ can be decomposed as the Hadamard product of two matrices $\mathbf{A}$ and $\mathbf{B}$, where $\mathbf{A}$ is a matrix characterized by the multipath amplitude correlation properties and $\mathbf{B}$ is a matrix characterized by the multipath ToA correlation properties. In conventional tapped-delay-line wideband MIMO channel models, the multipath ToA differences are neglected. Therefore all the entries of $\mathbf{B}$ are unity. However, once the multipath ToA differences in UWB MIMO channels are taken into account, the entries of $\mathbf{B}$ will have varying values from -1 to 1.

To further investigate the impact of ToA correlation on the SF channel covariance matrix $\mathbf{R}$, we consider a practical UWB MIMO system with $M_t = M_r = 2$, $\mathbf{d}_T = [0,\ 10]$ cm, $\mathbf{d}_R = [0,\ 20]$ cm, and $N = 128$. The total system bandwidth is set to be $N\Delta f = 3 \times 528$ MHz in accordance to current multi-band (MB) OFDM UWB standards. Matrix $\mathbf{B}$, which has real-valued entries, is then computed based on (18) and visualized in Fig. 1. The x-axis and y-axis represent the subcarrier indices following the same format as the channel vector $\mathbf{H}$ in (7). The 16 distinguishable grids in Fig. 1 reflect the $M_tM_r \times M_tM_r$ spatial structure. Inside each grid, the values change due to varying subcarrier frequencies. The grid at the upper left corner has entries with value 1 since the corresponding channel serves as the reference channel in terms of both antenna spacing and multipath ToA. From Fig. 1, it is observed that $\mathbf{B}$ has a significant impact on decorrelating the channels. This observation agrees with our previous findings in [7]. It is noted that when the system bandwidth increases, such a decorrelation effect becomes more significant. This is shown in Fig. 2 where the system bandwidth is changed to 7.5 GHz, corresponding to the maximum bandwidth available to UWB systems based on current regulations.

IV. Maximum Diversity Order Analysis

A structural SF channel covariance matrix can facilitate signal processing and information theoretic studies of UWB MIMO systems. In this paper, we give the maximum achievable diversity order analysis as an example. We consider SF coding across M_t transmit antennas and N OFDM subcarriers. Each codeword can be expressed as a $N \times M_t$ matrix [4]

$$\mathbf{C} = \begin{bmatrix} c^1(1) & c^2(1) & \cdots & c^{M_t}(1) \\ c^1(2) & c^2(2) & \cdots & c^{M_t}(2) \\ \vdots & \vdots & c^p(k) & \vdots \\ c^1(N) & c^2(N) & \cdots & c^{M_t}(N) \end{bmatrix} \quad (21)$$

where $c^p(k)$ is the channel symbol transmitted over the kth subcarrier by the pth transmit antenna. At the receiver, after matched filtering, removing the cyclic prefix, and applying the FFT, the received signal at the kth subcarrier and the qth receive antenna is given by

$$y^q(k) = \sqrt{\frac{\rho}{M_t}} \sum_{p=1}^{M_t} c^p(k) H^{p,q}(k) + z^q(k) \quad (22)$$

where ρ is the average signal-to-noise-ratio (SNR) and $z^q(k)$ denotes the additive white complex Gaussian noise with zero mean and unit variance at the kth subcarrier and the qth receive antenna. We can further write the receive signal as [4]

$$\mathbf{Y} = \sqrt{\frac{\rho}{M_t}} \mathbf{DH} + \mathbf{Z} \quad (23)$$

where the received vector $\mathbf{Y}$ of size $NM_r \times 1$ is given by

$$\mathbf{Y} = [y^1(0), \cdots, y^1(N-1), \cdots, y^q(k), \cdots, y^{M_r}(0), \cdots, y^{M_r}(N-1)]^T \quad (24)$$

$\mathbf{H}$ is the channel frequency response vector formatted as (7), and $\mathbf{Z}$ is the corresponding noise vector. In (23), $\mathbf{D}$ is a $NM_r \times NM_tM_r$ matrix constructed from the SF codeword $\mathbf{C}$ in (21) as follows

$$\mathbf{D} = \mathbf{I}_{M_t} \otimes \left[\mathbf{D}^1, \mathbf{D}^2, \cdots, \mathbf{D}^q, \cdots, \mathbf{D}^{M_r}\right] \quad (25)$$

where $\mathbf{I}_{M_t}$ is an identity matrix of size $M_t \times M_t$ and $\mathbf{D}^q = \mathrm{diag}\{c^q(0), c^q(1), \cdots, c^q(N-1)\}$ for any $q = 1, 2, \cdots, M_r$.

The maximum achievable diversity or full diversity is defined as the maximum diversity order that can be achieved by SF codes of size $N \times M_t$. In this paper, we only discuss the maximum diversity order as a upper bound of the maximum achievable diversity, without going into detailed discussion in whether the upper bound can be achieved or not. Suppose that $\mathbf{D}$ and $\tilde{\mathbf{D}}$ are two matrices constructed from two different codewords $\mathbf{C}$ and $\tilde{\mathbf{C}}$, respectively. The maximum diversity order Λ can then be determined as [4]

$$\Lambda = \mathrm{rank}\left\{(\mathbf{D}-\tilde{\mathbf{D}})\mathbf{R}(\mathbf{D}-\tilde{\mathbf{D}})^H\right\}. \quad (26)$$

According to the rank inequalities on Hadamard product and Kronecker product [8], we have

$$\Lambda \le \min\{NM_r, \mathrm{rank}(\mathbf{R})\}. \quad (27)$$

From the decomposition of $\mathbf{R}$ in (13), we get

$$\mathrm{rank}(\mathbf{R}) \le \mathrm{rank}(\mathbf{A})\mathrm{rank}(\mathbf{B}). \quad (28)$$

Since $\mathbf{A}$ can be further decomposed as shown in (14), it follows that

$$\begin{aligned}\operatorname{rank}(\mathbf{A}) &\leq \min\left\{\operatorname{rank}(\mathbf{I}_{M_r}\otimes\mathbf{W}),\operatorname{rank}(\mathbf{\Phi})\right\}\\ &\leq \min\left\{M_tM_r\min\left\{N,L\right\},\operatorname{rank}(\mathbf{\Phi})\right\}.\end{aligned} \tag{29}$$

In (29), $\mathbf{\Phi}$ is the $M_tM_rL \times M_tM_rL$ matrix given by (16). Applying matrix permutations to $\mathbf{\Phi}$ can yield another matrix

$$\tilde{\mathbf{\Phi}} = \mathbf{I}_L \otimes [\mathbf{R}_A(0),\mathbf{R}_A(1),\cdots,\mathbf{R}_A(L-1)] \tag{30}$$

where $\mathbf{R}_A(l)$ is defined in (2). From (30), it follows that

$$\operatorname{rank}(\mathbf{\Phi}) = \operatorname{rank}(\tilde{\mathbf{\Phi}}) \leq \sum_{l=0}^{L-1}\operatorname{rank}\left(\mathbf{R}_A(l)\right). \tag{31}$$

Substituting (28), (29), and (31) into (27), we get

$$\Lambda \leq \min\left\{NM_r, \operatorname{rank}(\mathbf{B})\sum_{l=0}^{L-1}\operatorname{rank}(\mathbf{R}_A(l))\right\}. \tag{32}$$

In case of wideband MIMO channel models (no ToA correlation), we have $\operatorname{rank}(\mathbf{B}) = 1$ and consequently

$$\Lambda \leq \min\left\{M_rN, \sum_{l=0}^{L-1}\operatorname{rank}\left(\mathbf{R}_A(l)\right)\right\} \tag{33}$$

which is in agreement with the result in [5]. However, for UWB MIMO channel models with ToA variations, from (12) and (18), it can be shown that $\operatorname{rank}(\mathbf{B}) \geq M_tM_r$. Here, the equality is fulfilled when $N = 1$. In addition, we have $\sum_{l=0}^{L-1}\operatorname{rank}(\mathbf{R}_A(l)) > L$. It follows that

$$\Lambda \leq \min\left\{M_rN, M_tM_rL\right\}. \tag{34}$$

This implies that once the ToA correlation/difference is taken into account, the total degrees of freedom available in a UWB MIMO channel are M_tM_rL, regardless of the multipath amplitude correlation properties.

V. Conclusions

In this paper, we have proposed a simple UWB MIMO channel model, which considers the spatial correlation of both multipath amplitudes and ToAs. We have shown that the SF channel covariance matrix $\mathbf{R}$ yielded by this model can be decomposed as the Hadamard product of two matrices $\mathbf{A}$ and $\mathbf{B}$, which are determined by the multipath amplitude correlation and ToA correlation, respectively. We have demonstrated that both matrices $\mathbf{A}$ and $\mathbf{B}$ have tractable structures, giving the proposed channel model an distinct advantage in facilitating theoretical analysis of UWB MIMO systems. To illustrate the usefulness of the proposed model, we analyzed the maximum diversity order of SF coded UWB MIMO systems. It has been shown that once the ToA correlation/variation is taken into account, each resolvable multipath component in a UWB MIMO channel will contribute to an additional degree of freedom to be exploited as diversity gains, regardless of the multipath amplitude correlation properties.

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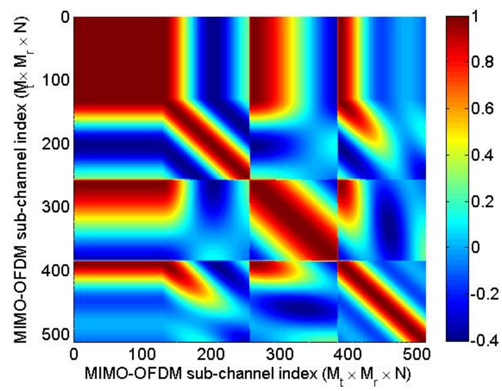

Fig. 1. Matrix $\mathbf{B}$ related to the multipath ToA correlation ($M_t = M_r = 2$, $\mathbf{d}_T = [0\ 10]$ cm, $\mathbf{d}_R = [0\ 20]$ cm, $N = 128$, and $N\Delta f = 3 \times 528$ MHz).

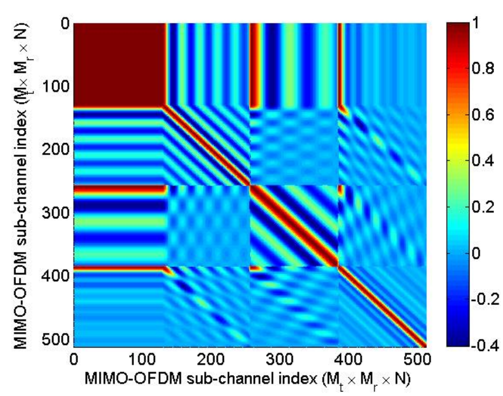

Fig. 2. Matrix $\mathbf{B}$ related to the multipath ToA correlation ($M_t = M_r = 2$, $\mathbf{d}_T = [0\ 10]$ cm, $\mathbf{d}_R = [0\ 20]$ cm, $N = 128$, and $N\Delta f = 7.5$ GHz).