This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2009 proceedings

Three Layered Hidden Markov Models for Binary Digital Wireless Channels

Omar S. Salih[1], Cheng-Xiang Wang[1], David I. Laurenson[2]
[1]Joint Research Institute for Signal and Image Processing, Heriot-Watt University, Edinburgh, EH14 4AS, UK.
[2]Joint Research Institute for Signal and Image Processing, University of Edinburgh, Edinburgh, EH9 3JL, UK.
Email: uss3@hw.ac.uk, cheng-xiang.wang@hw.ac.uk, dave.laurenson@ed.ac.uk

***Abstract*—Generative models are created to be used in the design and performance assessment of high layer wireless communication protocols and some error control strategies. Generative models can replace real digital wireless channels to significantly reduce the time and complexity of system simulation. The errors occurring in digital wireless channels are not independent but form clusters or bursts. Generative models have to produce error sequences having similar burst error statistics to those of original error sequences obtained from real digital systems. In this paper, we propose a generative hidden Markov model (HMM) with three layers. It is shown that the proposed three layered HMM can generate error sequences that have statistics compatible with those of original error sequences derived from an enhanced general packet radio service (EGPRS) transmission system.**

I. Introduction

A digital channel covers the entire transmission chain in a communication system embracing the physical channel together with the transmitter and receiver as long as the input and output are time-discrete. Errors occurring in digital wireless channels, due to various impairments, often consist of clusters or bursts. Comparing the input and output sequences of the digital channels yields error sequences. These sequences can replace the real digital channels for the purpose of dramatically reducing the computation burdens and simulation time. Models that describe reference error sequences (descriptive model) and are able to generate statistically similar error sequences (generative model) are called error models. They are substantial in the design and performance evaluation of error control strategies and high layer protocols [1].

The first generative model is based on Markov chains and consists of two states [2], one for the correct bits and the other for the erroneous bits. This model is elementary and hence produces error sequences having discrepant burst error statistics with those of the originals. To improve the performance of this model, the numbers of error and error-free states must be increased [3]. Simplified Fritchman Models (SFMs) [4] are well-known models which are considered as an improved version of two-states Markov models. They involve several error-free states and one erroneous state. Other generative models based on Stochastic Context Free Grammars (SCFGs) [5] and chaotic theory [6], [7] have also been developed in the literature. However, they are quite complex and created for specific types of error sequences. In addition, they do not show satisfactory results of burst error statistics. The most recent established generative models are called the deterministic process based generative models (DPBGMs) [8], [9]. They are the best generative models so far in generating error sequences having accurate burst error statistics compared with original error sequences. However, a drawback of the DPBGMs is that they retrieve the error bursts directly from the original error sequences rather than create new error bursts in the process of generating error sequences. This would limit generating other error bursts and consequently error sequences when the channel conditions vary.

HMMs [10] have become applicable for modeling many stochastic processes and sequences e.g., pattern recognition [11]. In HMMs, the modeled system is considered as a Markov process with unknown parameters. The main task is to determine the hidden parameters from some observations. The Baum-Welch algorithm is a common procedure that trains the observations in order to find the hidden parameters. Baum-Welch based HMMs (BWHMMs) [12] give a justifiable behavior for error sequences with error bursts of bell-shaped error density. However, other kinds of error sequences are hard to characterize using such models. Hierarchal [13] and Layered HMMs (LHMMs) [14] have been introduced for the purpose of dramatically reducing the amount of training data required for them to achieve a performance comparable to conventional HMMs. In [15], we have proposed two layered HMMs named as double embedded processes based HMMs (DEPHMMs). The first layer is composed of only one error-free state and several error burst states classified according to their maximum gap length, whereas the second layer constructs error bursts inside the error burst states. For this model, it is difficult to determine the number of error burst states that gives a good burst error statistics approximation to the reference ones.

In this paper, we propose a novel generative model, called three layered HMMs (3LHHM). The first layer is made up of one error-free burst state and several error burst states assigned according to the maximum error cluster lengths. The second layer divides further the classes of error bursts based on their maximum gap lengths. The final layer constructs the error bursts. Implementing the 3LHMMs and determining its parameters are simple and straightforward. Binary bit error sequences are used and the resulting burst error statistics are competent.

This paper is organized as follows. Section II introduces some terms and definitions related to the binary bit error sequences and their statistics. The structure and parameterization of the 3LHMM are demonstrated in Section III. In Section IV, the simulations of the burst error statistics related to the 3LHMM are shown and compared with those of the original error sequence obtained from an EGPRS transmission system and other generative models. Conclusions are drawn in Section V.

II. Terms and Definitions

A binary error sequence $\{e_j\}$ is obtained by carrying out modulo 2 addition between input sequence to a digital wireless channel $\{i_j\}$ and the output sequence of the channel $\{o_j\}$. That is $e_j = i_j \oplus o_j$, where $i_j, o_j \in \{0,1\}, j = 1, 2, ...$ and $\oplus$ is a modulo 2 addition operator. Clearly, a binary error sequence is a sequence of "0"s and "1"s, where "0" denotes a correct bit and "1" denotes an error bit. The error sequence is identified by many terms as follows.

A *gap* is described as a series of consecutive zeros between two ones. Its length equals the number of zeros. An *error cluster* is a series of consecutive ones having a length equal to the number of ones. In order to distinguish between some long gaps and the other gaps which are considered as part of the error bursts, a threshold value η can be assigned. This turns out two new definitions: error-free burst and error burst. An *error-free burst* is a sequence of consecutive zeros that have at least η bits in length. Note that an error-free burst can in general be considered as a gap with a minimum length restriction, except that an error-free burst is not necessarily located between two ones. An *error burst* is a sequence of zeros and ones delimited by two ones. It is separated from other error bursts by error-free bursts. The minimum length of an error burst is 1. Note that the error sequences are made up of successive error-free bursts and error bursts, or successive gaps and error clusters. On the other hand, error bursts are also recognized by successive gaps with lengths less than η and error clusters.

For the purpose of extensive analysis of error sequences, many burst error statistics have been elaborated in the literature [9]. These statistics are also used to scrutinize the performance of generated error sequences. The burst error statistics that will be used in this paper are listed below:

1) $P(0^{m_0}|1)$: error-free run distribution (EFRD), which is the probability that an error is followed by m_0 or more error-free bits [4].
2) $G(m_g)$: gap distribution (GD), which is the cumulative distribution function (CDF) of gap lengths m_g [4], [9].
3) $P(1^{m_c}|0)$: error-cluster distribution (ECD), which is the probability that a correct bit is followed by at least m_c successive error bits [4].
4) $P(m,n)$: block error probability distribution (BEPD), which is the probability that a block of n bits contains at least m errors [1].
5) $Q(l,n)$: block burst probability distribution (BBPD), which is the probability of an error burst of length l occurring in a block of length n. For only this statistic, the length of a burst in a block of n digits is the number of zeros and ones between the first error and the last error in the block (both errors included) regardless of the nature of the digits in between [12].
6) $\rho(\Delta k)$: bit error correlation function (BECF), which is the conditional probability that the Δkth bit following an error bit is also in error [9].

III. The proposed Generative Model

In order to distinguish the error-free bursts from the error bursts, the value of η should be determined. It can be chosen through a range of values between η_i and η_f so that the error burst identification is not affected. These values can be obtained from the EFRD curve when it is flat.

The 3LHMM design is shown in Fig. 1. One state is used to represent the error-free bursts because they consist of "0"s only. On the contrary, the error-bursts have various structural configurations, therefore, they can be entitled to classification. Thus, several error burst states are required, each state typifies a common structural behavior. The classification criterion for the first layer is the maximum error cluster length in each error burst. This means that each class is represented by one state and each state contains all error bursts that have the same maximum error cluster length. The number of values of the maximum error cluster lengths in the original error sequence determines the number of error burst states in the first layer. For the purpose of further classification, each error burst state in the first layer can be partitioned into internal states according to the maximum gap lengths of its error bursts. The internal states have error bursts within equal intervals of the entire maximum gap lengths range of each error burst state. The maximum gap lengths in each state have long range, consequently, the number of internal states can be large, e.g., 8-15. Those internal states are held to be the second layer in the model. The third layer is dedicated to constructing error bursts. Clearly, an error burst consists of error clusters of different lengths and gaps of lengths less than η. Since the errors are our concern in error models, we allocate several substates of the second layer for the error clusters in a manner that each substate represents one error cluster length. Moreover, because the gaps of the error bursts consist of "0"s only, one state is still sufficient to represent them. Generating gaps of length less than η in the third layer and error-free bursts in the first layer depends only on their lengths distribution.

The parameters of the 3LHMM are as follows:

1) N: the number of error burst states in the first layer, i.e., $S = \{s_1, s_2, ..., s_N, s_{N+1}\}$, where S is the set of states and N is the number of the values of the maximum error cluster lengths in the original error sequence.
2) M_p: the number of the internal states for each error burst state, i.e., $W_p = \{w_{1_p}, w_{2_p}, ..., w_{M_p}\}$, where W_p is the set of internal states and $p = 1, ..., N$. The parameter M_p is chosen to be large enough e.g., 8-15.
3) $L_{p,q}$: the number of error clusters substates in each internal state, i.e., $V_{p,q} = \{v_{1_{p,q}}, v_{2_{p,q}}, ..., v_{L_{p,q}}, v_{L_{p,q}+1}\}$, where $V_{p,q}$ is the set of substates and $q = 1, ..., M_p$. The

parameter $L_{p,q}$ is designated according to the number of error cluster lengths in each internal state.

4) $\mathbf{F} = (f_{i,j})$: the state transition matrix, where $f_{i,j}$ is the transition probability from s_i to s_j, such that

$$f_{i,j} = P\left[Q_{t+1} = s_j \,|\, Q_t = s_i\right], 1 \le i, j \le N+1$$
$$= \begin{cases} 1, & 1 \le i \le N, j = N+1, \\ \frac{N_{EB,j}}{\sum_{j=1}^{N} N_{EB,j}}, & i = N+1, 1 \le j \le N, \\ 0, & \text{otherwise}, \end{cases} \quad (1)$$

with Q_t being the current state at time t and $N_{EB,j}$ the number of error bursts in s_j. The state transition matrix is

$$\mathbf{F} = \begin{pmatrix} 0 & \cdots & 0 & 1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 1 \\ f_{N+1,1} & \cdots & f_{N+1,N} & 0 \end{pmatrix}. \quad (2)$$

5) $\mathbf{D}_{p,q} = ((d_{h,k})_{p,q})$: the substate transition matrix, where $(d_{h,k})_{p,q}$ is the transition probability from $v_{h_{p,q}}$ to $v_{k_{p,q}}$, such that

$$(d_{h,k})_{p,q} = P\left[R_{t+1} = v_{k_{p,q}} | R_t = v_{h_{p,q}}\right], 1 \le h_{p,q}, k_{p,q} \le L_{p,q}+1$$
$$= \begin{cases} 1, & 1 \le h_{p,q} \le L_{p,q}, k_{p,q} = L_{p,q}+1, \\ \frac{N_{C,k_{p,q}}}{\sum_{k_{p,q}=1}^{L_{p,q}} N_{C,k_{p,q}}}, & h_{p,q} = L_{p,q}+1, 1 \le k_{p,q} \le L_{p,q}, \\ 0, & \text{otherwise}, \end{cases} \quad (3)$$

with R_t being the current state at time t and $N_{C,k_{p,q}}$ the number of error clusters in $k_{p,q}$. The substate transition matrix is

$$\mathbf{D}_{p,q} = \begin{pmatrix} 0 & \cdots & 0 & 1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 1 \\ d_{L_{p,q}+1,1} & \cdots & d_{L_{p,q}+1,L_{p,q}} & 0 \end{pmatrix}. \quad (4)$$

6) $\mathbf{A} = (a_j(n))$: the first layer emission probability distribution matrix, where $a_j(n)$ $(1 \le j \le N+1)$ is the probability of getting the burst x_n in state s_j, that is

$$a_j(n) = P\left[x_n \text{ at } t | Q_t = s_j\right], 1 \le n \le N_{EB,j}, N_{EFB,j}.$$

$N_{EFB,j}$ is the number of error-free bursts in s_j.

7) $\mathbf{B} = (b_u(m))$: the second layer emission probability distribution matrix, where $b_u(m)$ is the probability of getting the error burst y_m from the internal state w_{1_p}, that is

$$b_u(l) = P\left[y_m \text{ at } t | T_t = w_{1_p}\right], 1 \le m \le M_p,$$

where T_t is the current state at time t.

8) $\mathbf{E} = (e_{k_{p,q}}(l))$: the third layer gap emission probability distribution matrix, where $e_{k_{p,q}}(l)$ $(k_{p,q} = L_{p,q}+1)$ is the probability of getting the gap z_l in state $v_{k_{p,q}}$, that is

$$e_{k_{p,q}} = P\left[z_l \text{ at } t | R_t = v_{k_{p,q}}\right], 1 \le l \le N_{G,k_{p,q}}.$$

$N_{G,k_{p,q}}$ is the number of gaps in $k_{p,q}$.

9) $\mathbf{\Pi}_{p,q} = ((\pi_k)_{p,q})$: the initial substate distribution vector, where $(\pi_k)_{p,q}$ is the probability of $v_{k_{p,q}}$ to be an initial substate.

$$\mathbf{\Pi}_{p,q} = \left(d_{L_{p,q}+1,1}, ..., d_{L_{p,q}+1,L_{p,q}}, 0\right), \quad (5)$$

which assures the initiation of an error burst by an error cluster, otherwise the definition of error burst is no longer valid. Similarly, we can obtain the termination substate distribution vector $\mathbf{\Omega}_{p,q}$, which ensures that the error burst is ending with an error cluster, and the initial state distribution vector.

10) $\delta_{n,p,q}$: error burst length values. These values regulate the termination of error burst lengths, so that $\mathbf{\Omega}_{p,q}$ shall be activated according to them. The Activation takes place when the generated error burst lengths become either equal or around a chosen δ_n. The deviation from δ_n shall be small enough, otherwise, the current generated error burst shall be discarded and the process shall be repeated again. The $\delta_{n,p,q}$ are acquired from the reference error burst lengths distribution.

11) Γ: the length of the generated error sequence. This value terminates the error sequence generation once it is reached or exceeded regardless the current state is error burst or error-free burst.

The above mentioned parameters set up the 3LHMM. In summary, each substate in the third layer of the 3LHMM creates error bursts and forward them to the second layer, which in turn collects all the created error bursts and forward them to the first layer. The first layer combines the error bursts with the error-free bursts to produce error sequences. But, before generating error sequences, the produced error bursts in each state should be tested to ensure that they convey similar statistical behavior to the originals. The testing parameters in each state are the mean value of error cluster lengths and mean value of the gap lengths of the error bursts. If the matching test fails, it can be conducted to the internal states of the failed state. The internal state which fails the test must be divided into several parts. If this procedure does not work, then 10-50 % of the related gaps in the failed internal state should be deleted since many of them are duplicate.

IV. Simulation Results and Discussions

For the sake of model parametrization, a reference error sequence should be obtained from a real system. We have used an uncoded EGPRS transmission system with ideal frequency hopping. The wireless digital channel is composed of a Gaussian minimum shift keying (GMSK) modulator and demodulator, a typical urban (TU) channel with noise and interference, and a hard decision Viterbi equalizer. The mobility speed is 3 km/h. The data are transmitted using time-division multiple access (TDMA) with blocks of 116 bits and transmission rate of $F_s = 270.8$ kb/s.

The chosen reference error sequence has 15 million bits and is corresponding to a carrier-to-interference ratio (CIR) of 8 dB. It exhibits long error bursts interleaved by long error-free bursts. We find η from Fig. 2 which presents the EFRD.

From its shape plateau, we find $\eta_i = 400$ and $\eta_f = 1000$ hold. The chosen value of η is 800. The value of N is 19, whereas M_p is fixed to 10 and $\Gamma = 20$ million bits.

For the purpose of comparison, a SFM, BWHMM, and DPBGM are implemented. The parameters of a SFM with K states are obtained by fitting the weighted sum of $K-1$ exponentials to the EFRD. The number of states used for the SFM is 6. No better improvement of the SFM statistics could be accomplished by increasing this number of states. In the BWHMM, the number of classes is 12, whereas the total number of states is 400. The number of bits which should represent each block in the error sequence is chosen to be 103 bits. For the DPBGM, the vector $\Psi = (9, 10, 0.09, 0.0783, 73.22Hz, 0.8132ms)$, $R_B = 0.9344$, and $q_s = 0.01$ hold.

In order to appraise the 3LHMM, we have to inspect how close its burst error statistics can fit those of the reference error sequence. Figs. 2-7 demonstrate the behavior of the burst error statistics, mentioned in Section II, for an EGPRS reference error sequence and others attained from well-known generative models mentioned before. It is shown that the SFM fails to describe the ECD, BEBD, BBPD, and BECF statistics, whereas the BWHMM has better description for them than the SFM except for the EFRD and GD. On the other hand, the BWHMM burst error statistics performance still does not reach the one of the reference error sequence. This remark excludes the BBPD which has good behavior for block bursts of lengths 0-60 bits. The lack of agreement is because that the BWHMM behavior was conceived to best characterize the bell-shaped error density bursts. But, our reference error sequence has many error bursts which do not conform to such a shape. However, the EFRD, GD, ECD, BEBD, BBPD, and BECF statistics of the DPBGM have small differences from the 3LHMM statistics which nearly match the reference sequence statistics. On the other hand, the 3LHMM illustrates perfect match to the reference sequence for the BBPD. Therefore, the 3LHMM leads the other generative models knowing that the DPBGM retrieves the error bursts from the reference error sequence rather than constructs them by itself.

V. Conclusions

This paper has developed a three layered HMM for binary error sequences. The generated error sequences by this method are comparable to those obtained from an EGPRS system. This is because that the 3LHMM is able to create thorough error bursts. This could not be achieved by the SFM and BWHMM. The SFM can best characterize the EFRD and GD only, whereas, the BWHMM is good at characterizing the BBPD but for small values of block burst lengths. The DPBGM does not create error bursts and failed to typify the BBPD. However, it demonstrated a satisfactory agreement with the EGPRS regarding the other burst error statistics. Subsequently, the 3LHMM leads the SFM, BWHMM, and DPBGM.

Acknowledgment

We acknowledge the support from the Scottish Funding Council for the Joint Research Institute in Signal and Image Processing between the University of Edinburgh and Heriot-Watt University which is a part of the Edinburgh Research Partnership in Engineering and Mathematics (ERPem). The authors would also like to acknowledge gratefully the sponsorship of this work by the EPSRC and Philips Research Laboratories (PRL), Cambridge, UK. We also thank Mr. Rob Davies from PRL for helpful discussions.

References

[1] M. Zorzi and R. R. Rao, "Perspectives on the impact of error statistics on protocols for wireless networks," *IEEE Personal Commun.*, vol. 6, no. 10, pp. 32–40, Oct. 1999.

[2] E.O. Elliot, "Estimates of error rates for codes on burst-noise channels," *Bell Syst. Tech. J.*, vol. 42, pp. 1977–1997, Sept. 1963.

[3] Y. Yu, S. L. Miller, "A four-state Markov frame error model for the wireless physical layer," in *Proc. IEEE WCNC'07*, Hong Kong, Mar. 2007, pp. 2053–2057.

[4] B. D. Fritchman, "A binary channel characterization using partitioned Markov chains," *IEEE Trans. Information Theory*, vol. 13, no. 2, pp. 221–227, Apr. 1967.

[5] W. Zhu and J. Garcia-Frias, "Stochastic context-free grammars and hidden Markov models for modeling of bursty channels," *IEEE Trans. Veh. Technol.*, vol. 53, no. 3, pp. 666–676, May 2004.

[6] E. Costamagna, L. Favalli, and P. Gamba, "Multipath channel modeling with chaotic attractors," *Proc. of the IEEE*, vol. 90, no. 5, pp. 842–859, May 2002.

[7] A. Köpke, A. Willig, and H. Karl, "Chaotic maps as parsimonious bit error models of wireless channels," *Proc. IEEE INFOCOM'03*, San Francisco, USA, Mar. 30–Apr. 3, 2003, pp. 513–523.

[8] C.-X. Wang and M. Pätzold, "A generative deterministic model for digital mobile fading channels," *IEEE Commun. Letters*, vol. 8, no. 4, pp. 223–225, Apr. 2004.

[9] C.-X. Wang and W. Xu, "A new class of generative models for burst-error characterization in digital wireless channels," *IEEE Trans. Comm.*, vol. 55, no. 3, pp. 453–462, March 2007.

[10] W. Turin, *Digital Transmission Systems: Performance Analysis and Modeling*. New York: McGraw-Hill, 1999.

[11] I. Shahin, "Talking Condition Identification Using Second-Order Hidden Markov Models," *Proc. ICCIA'08*, Perugia, Italy, April 2008, pp. 1–6.

[12] J. Garcia-Frias and P. M. Crespo, "Hidden Markov models for burst error characterization in indoor radio channels," *IEEE Trans. Veh. Technol.*, vol. 46, no. 6, pp. 1006–1020, Nov. 1997.

[13] H. H. Bui, D. Q. Phung, S. Venkatech"Hierarchical Hidden Markov Models with General State Hierarchy," *Proc. AAAI'04*, San Jose, CA, USA, Jul. 2004, pp. 1–6.

[14] N. Oliver, A. Garg, E. Hovitz, "Layered representations for learning and inferring office activity from multiple sensory channels," *Comp. vision and Img. understanding*, vol. 96, pp. 163–180, Feb. 2004.

[15] O. S. Salih, C.-X. Wang, D. I. Laurenson,"Double embedded processes based hidden Markov models for binary digital wireless channels," *Proc. ISWCS'08*, Reykjavik, Iceland, Oct. 2008, pp. 219–223.

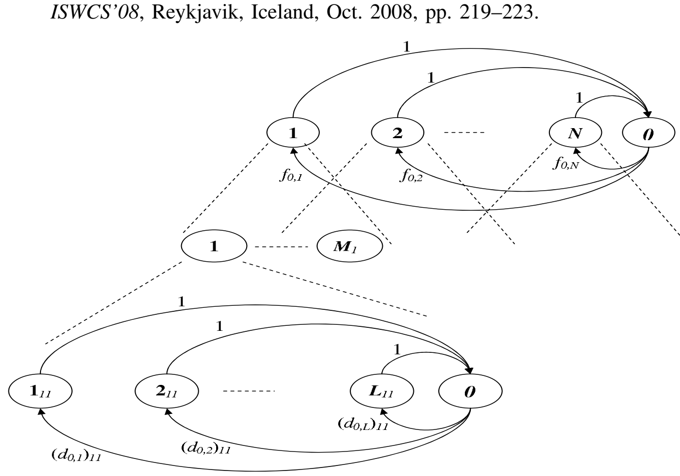

Fig. 1. The 3LHMM.

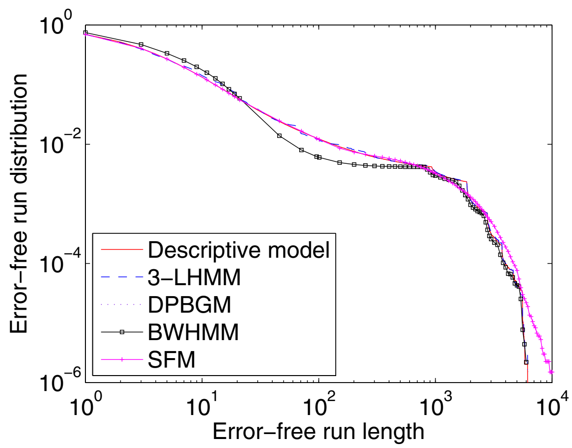

Fig. 2. EFRDs of the descriptive model obtained from the EGPRS system and different generative models.

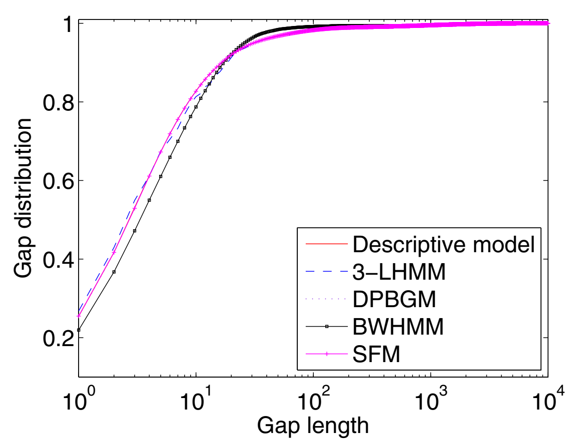

Fig. 3. GDs of the descriptive model obtained from the EGPRS system and different generative models.

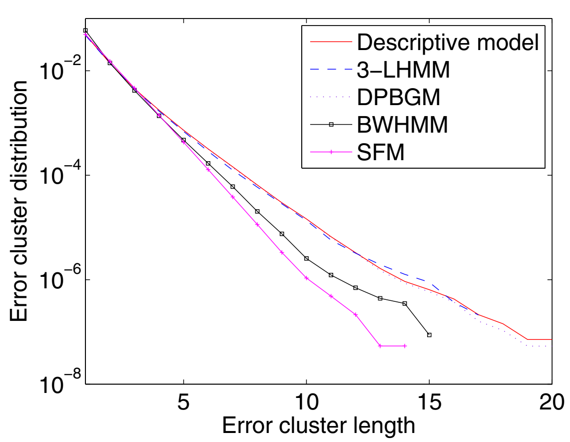

Fig. 4. ECDs of the descriptive model obtained from the EGPRS system and different generative models.

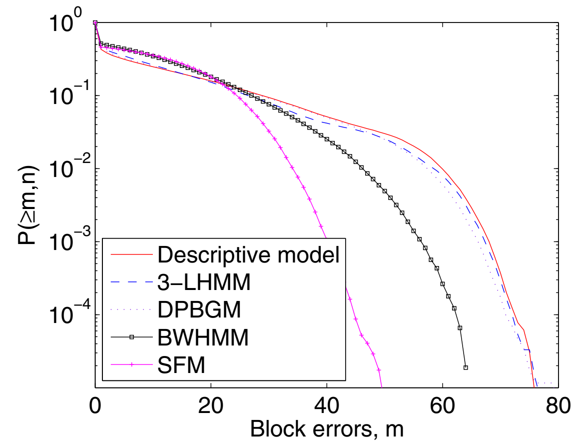

Fig. 5. BEPDs of the descriptive model obtained from the EGPRS system and different generative models ($n = 116$).

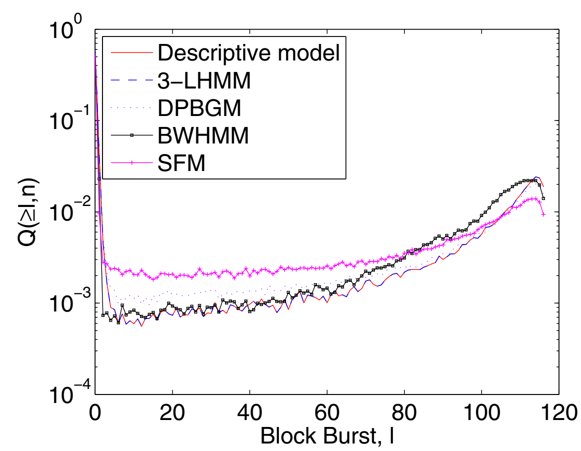

Fig. 6. BBPDs of the descriptive model obtained from the EGPRS system and different generative models ($n = 116$).

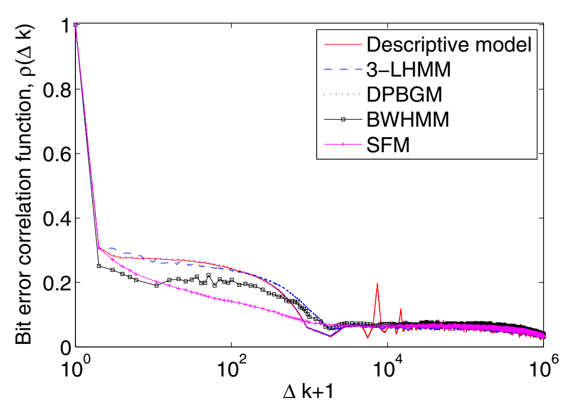

Fig. 7. BECFs of the descriptive model obtained from the EGPRS system and different generative models.