Heriot-Watt University
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Systems and Information Theory 5 – Digital Communication Systems

5.1 Introduction and Objectives:

Digital communication systems possess many advantages: this section covers some of the other signal processing operations required before a signal may be transmitted as a *pulse-code-modulated* or “PCM” signal.

At the end of this topic you will be able to:

State the advantages of digitising a signal
Calculate the sampling rate required
Explain the consequences of under sampling
Choose the cut-off frequencies of anti-alias and reconstruction filters
Compare a digital channel using repeaters with an analogue channel using amplifiers
Explain what is meant by time-division multiplexing
Describe the main characteristics of a PCM telephone heirachy

5.2 Reading list

You will find this material in many elementary comms textbooks, including the following

Usher and Guy, "Information and Communication for Engineers"
Glover and Grant, "Digital Communications"
Petersen, "Audio, Video and Data Telecommunications

5.3 The advantages of pulse-code modulated (PCM) communication systems

Many signal sources are analogue in nature, eg speech and video, but the signal is transmitted or stored as a digital signal. PCM signals have the following advantages over analogue:

- Easier to store as a pattern of 1's and 0's
- Easier to process in computers and ***digital signal processors***
- Can be coded for security and error correction purposes
- Several digital signals can easily be interleaved (multiplexed) and transmitted on one channel
- Noisy digital signals can be regenerated more effectively than analog signals can be amplified.

However, an analog signal must first be digitised: there are two distinct processes involved, the ***sampling*** of the signal, and the conversion from ***analog to digital***.

5.4 Sampling of an analog signal

It is important that the signal is sampled fast enough to avoid the loss of information, but not so often to result in too many unnecessary samples.

5.4.1 Natural sampling

Consider the system in **Fig 5.1a** where the switch **(ii)** periodically closes for τ seconds and samples the analogue signal $\mathbf{v_i(t)}$ every **T** seconds, giving a sampling rate of $\mathbf{f_s = 1/T}$ **samples/second**:

The output consists of narrow slices of the analogue signal spaced apart by T seconds **(iii)**: these samples are digitised and coded for transmission **(iv)** in an ***Analogue-to-Digital Converter*** (ADC).

At the output of the channel, a ***Digital-to-Analogue Converter*** (DAC) produces a replica of the sampled signal from the digital code **(v)**. The reconstruction filter **(ix)** then “smooths” the sampled signal to remove the effects of the narrow-pulse sampling. In order for this to occur without losing part of the signal the sampling frequency and the bandwidth of the filter must satisfy certain conditions.

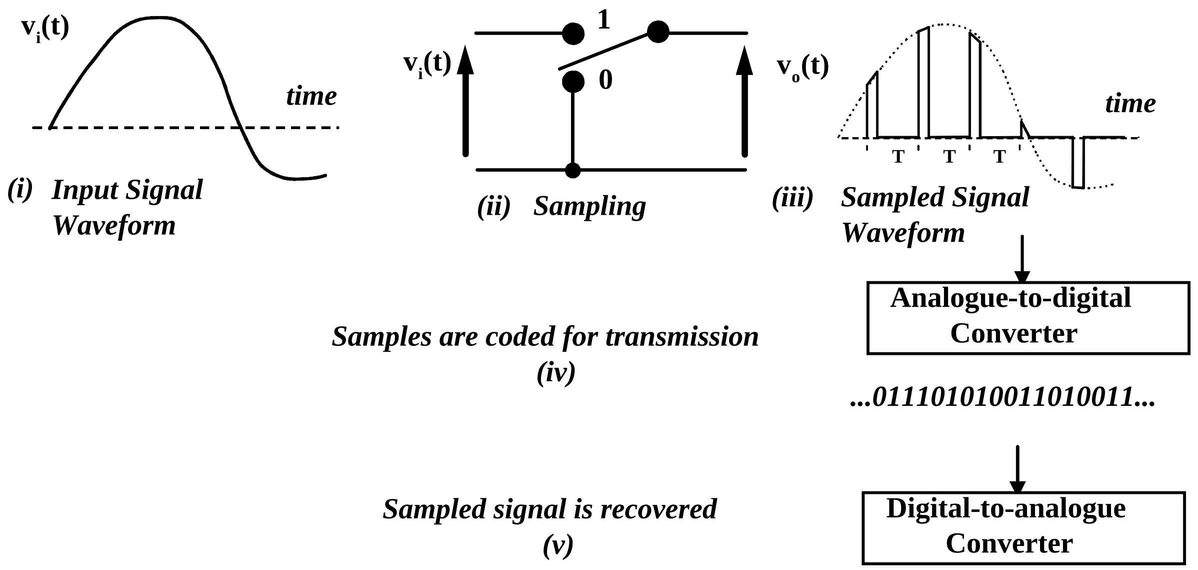

Fig 5.1a Digitising an analogue signal

The width of the sampling pulse is not of prime importance as far as the sampling process is concerned. However, the ADC will require the sample to be held constant until it has been converted. This is done with a ***Sample-and-hold*** circuit. The sampled value is held constant until conversion is complete, or the next sample is taken.

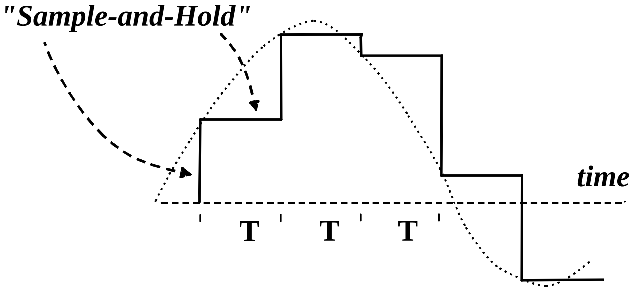

Fig 5.1b Sample-and-hold

5.4.2 Reconstruction of the signal

Sampling is equivalent to the ***multiplication*** of a signal by a train of (0,1) pulses with width τ and period T **(vi)**.

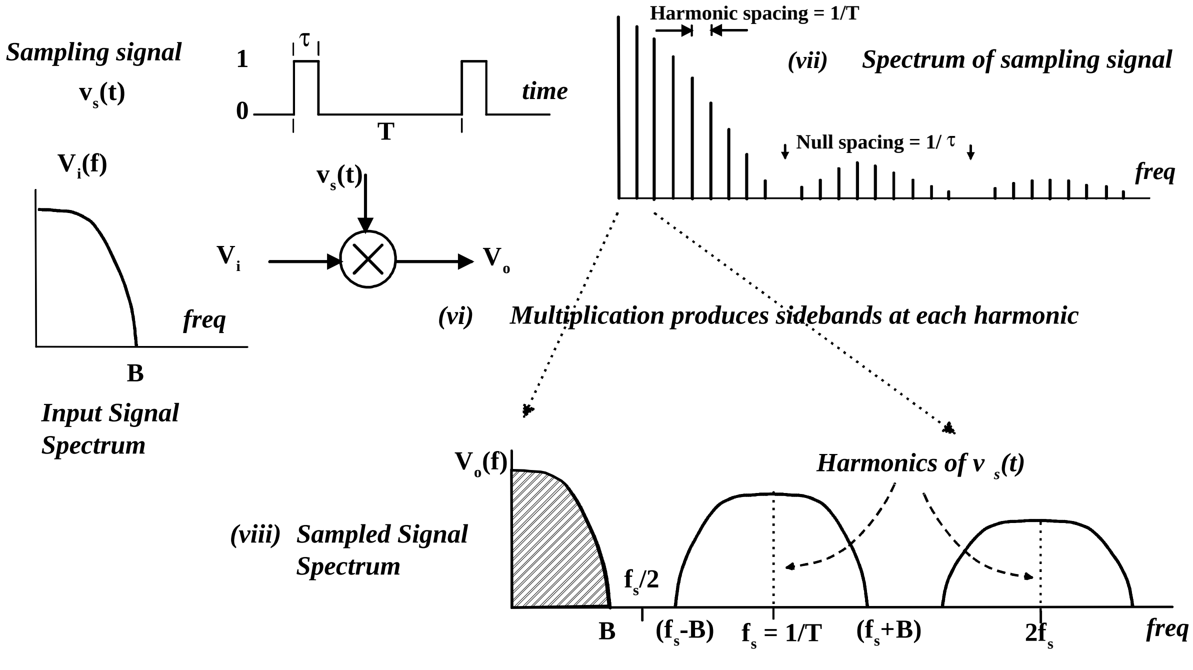

Fig 5.1c Spectrum of a sampled signal

As we know, multiplication produces **Amplitude Modulation** which generates sidebands around a "carrier" frequency. As we have seen[1], a pulse waveform has a spectrum **(vii)** consisting of a dc level, the fundamental frequency **$f_s = 1/T$** and its harmonics all of which are separated by the fundamental frequency **$1/T$**. (**Fig 5.1c**)

- Every one of these harmonics will be modulated by the sampled signal, so it follows that the spectrum of the sampled signal **($V_s(f)$)** consists of the original signal spectrum *together with sidebands around every harmonic of the sampling signal.* **(viii)**
- If the original signal has a bandwidth **B** herz, then each sideband will extend **B** herz on either side of each harmonic.

5.4.3 Recovery of the signal and the minimum sampling rate

It is easily seen that we can recover the original analogue signal with a suitable filter ***as long as the sampling sidebands do not overlap*** **(Fig 5.1c)**. For this to be the case, we require **$f_s/2 > B$**, where the original signal has a bandwidth **B** herz. Thus the sampling rate **$f_s = 1/T$** must be at least equal to **2B**. This gives us the ***Nyquist Sampling Theorem***:

For a signal of bandwidth B herz, the minimum sampling rate is 2B samples/s

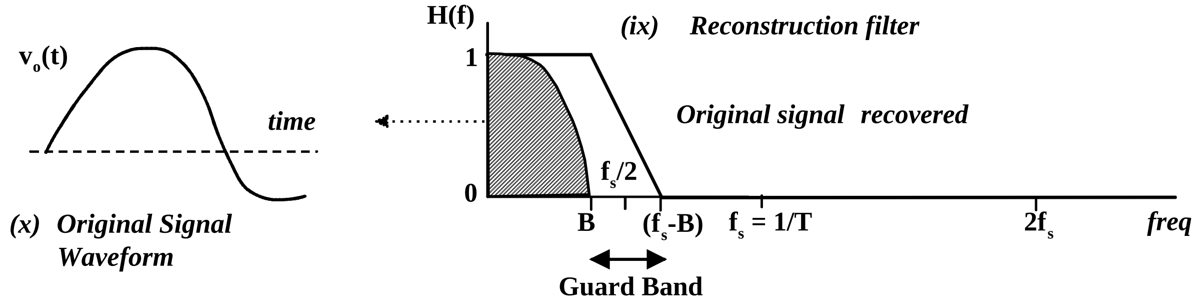

Fig 5.1d Recovery of the sampled signal

Since the filter recovers the original analog signal **(x)** from the samples it is called the ***reconstruction filter*** or the ***smoothing filter***. Of course, a practical filter cannot have a sudden cut-off, (and sharp cut-off filters usually distort the phase of a signal) so it is necessary to have a ***guard band*** separating the harmonics (**Fig 5.1d**), and the practical sampling rate must be greater than the Nyquist rate.

- For the compact disc (Audio CD) the maximum signal frequency is 20 kHz and the sampling rate is 44.1 kHz. Hence the guard band is 4.1 kHz wide.
- In the telephone system (see Section 5.8), the speech signal has a bandwidth up to 3.4 kHz and a sampling rate of 8 kHz, so the guard band is 1.2 kHz wide. Sampling at a rate higher than the Nyquist rate is called ***oversampling***.

5.4.4 Undersampling and aliasing distortion

If the sampling rate is too low for the bandwidth then the lower sideband will overlap the baseband and produce high frequency distortion known as ***aliasing***.

High frequency signal components are translated down the spectrum as if they have been reflected about the sampling frequency and therefore have a different frequency or "alias".

The effect can be seen in **Fig 5.2** for a single frequency sinewave (which, of course, has a single frequency component for the fundamental and each sampling sideband). With undersampling, there are insufficient samples to define the original signal unambiguously, and the reconstructed signal has a frequency given by the alias.

It is also called ***foldback*** distortion (**Fig 5.3**), since reflection of the higher frequency components appears to “fold” the spectrum back on itself.

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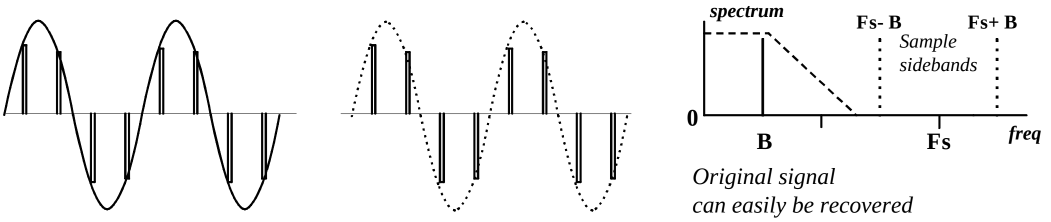
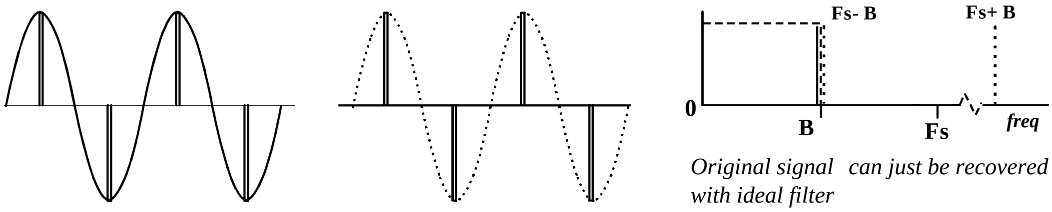
[1] See the Digital Oscilloscope Experiment, and Appendix A1



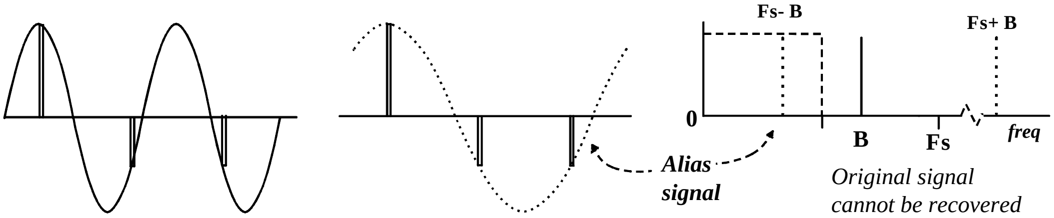
Original signal
can easily be recovered

(i) Oversampled - $F_s = 4B$



Original signal can just be recovered
with ideal filter

(ii) Nyquist sampling - $F_s = 2B$



Original signal
cannot be recovered

(iii) Undersampling - $F_s = 1.5B$

Fig 5.2 Aliasing distortion

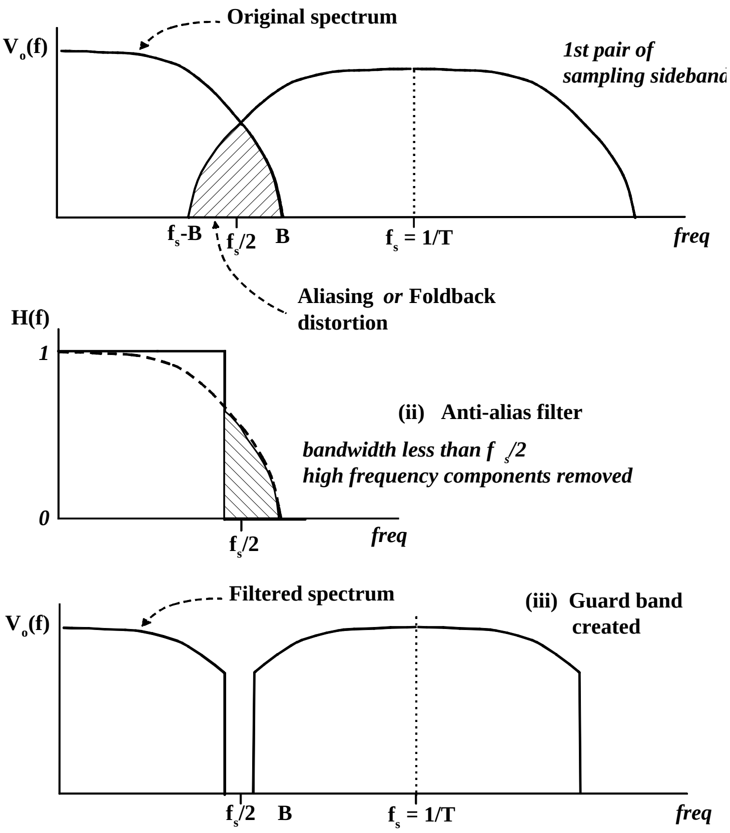
5.4.5 The Anti-alias (Pre-sampling) filter

In practice a signal will usually extend someway beyond its essential bandwidth, and will always be filtered by an ***anti-alias (or pre-sampling) filter*** before it is sampled. This will be a low-pass filter having a cut-off frequency somewhat less than the Nyquist frequency $f_s/2$ in order to create a guardband.

The anti-alias filter will necessarily remove the higher frequencies from the signal: if this cannot be tolerated, then the sampling frequency must be increased.

As noted above, the digital telephone system is limited to a maximum frequency of 3.4kHz and sampled at 8kHz, giving a guard band of 1.2kHz

(i) Signal bandwidth > $f_s/2$



(ii) Anti-alias filter

bandwidth less than $f_s/2$
high frequency components removed

(iii) Guard band created

Fig 5.3 The anti-alias filter

5.5 Regeneration of digital signals

All signals will be attenuated during transmission, and the **Signal-to-Noise Ratio (SNR)** will therefore fall as the transmission distance increases. The maximum transmission distance will be determined by the minimum allowable SNR, which will vary with the type of signal.

A long transmission path can be divided into sections and amplifiers inserted to compensate for the attenuation. However, it is not possible to remove all the noise from a signal even with very efficient filtering, and the remaining noise will be amplified along with the signal so that the SNR continues to decrease along the channel

Digital channels offer the possibility of *periodic reconstruction* of the original signal by **repeaters** inserted at the end of each section of the transmission path, and as long as the signal to noise ratio has not fallen too far in each section each replica will be almost free from errors.

This means that long distance digital channels can give better quality than analog channels if repeaters can be inserted into the transmission path, as is the case with cables and point-to-point microwave links.

5.5.1 Signal-to-Noise comparison of analog and digital channels

(i) The analog channel

Let the analogue channel have **k** identically noisy sections each with attenuation **A** (and amplifiers with gain **A** to compensate for the loss).

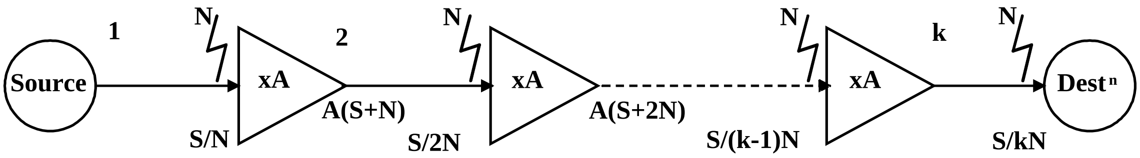
If the SNR at the input to the first amplifier is **S/N**, it can be seen that the *noise accumulates along the channel* and the SNR at the output is **S/kN**.

(ii) The digital channel

In contrast, a digital channel with ideal repeaters could *regenerate an error-free signal* for the next section so that the noise does not accumulate, and would have an output SNR of **S/N**, exactly the same as for a single section, giving an improvement of **$10\log_{10}k$ decibels** with respect to the ananlog channel.

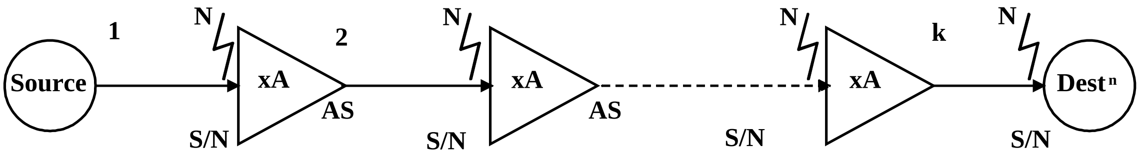
In practice there will be a small cumulative error rate produced by the noise in each section of the digital channel, but it can be shown that this is still far less than in the analogue channel.

(i) Analogue Channel



Attenuation A in each section, Gain A in each amplifier ***Output SNR***

(ii) Digital Channel



Attenuation A in each section, Gain A in each repeater ***Output SNR***

Fig 5.4 Repeaters and Amplifiers

5.6 A digital communication system - "PCM"

A typical digital communication system will therefore contain the following components **(Fig 5.5)**:

- Anti-alias Filter*
- Digitiser/Sample-and-Hold circuit*
- Analogue-to-Digital Converter*
- Coding-
    - ***Source*** *coding for data compression,*
    - ***Line*** *coding for signalling efficiency*
    - ***Error*** *coding to reduce the effect of errors*
- Modulator
- Physical Channel (with repeaters if necessary)*
    - *Copper cables*
    - *Fibre Optic cables*
    - *Radio*[2]
    - *Sonar*
    - *Recording medium*
- Demodulator
- Decoder (Source-, Line- and Error-)
- Digital-to-Analogue Converter*
- Reconstruction Filter*

* These components are essential: the others are not always required.

When used for telephone signals this system is referred to as ***Pulse-Code Modulation*** or "PCM"[3], and similar schemes are used for digital radio and television, mobile phone networks, CD/DVD recording, etc.

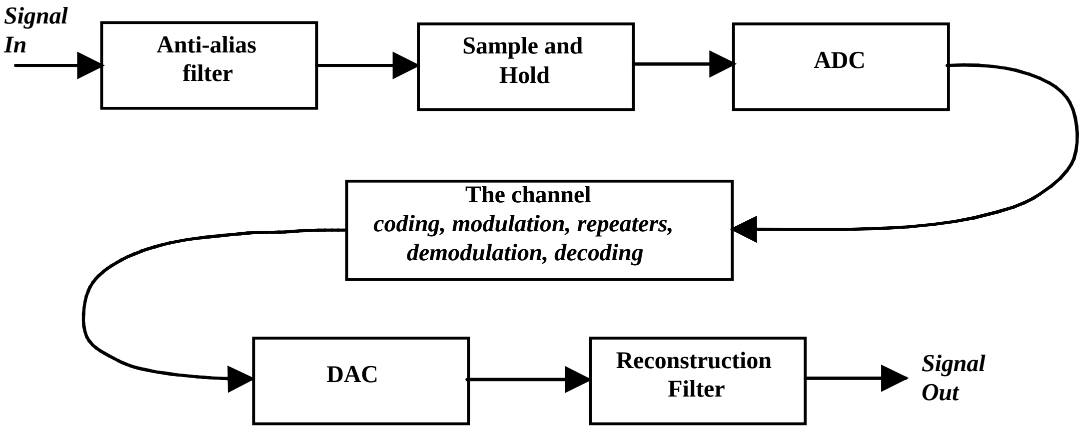


Fig 5.5 A digital Communication System

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[2] "Radio" is the general term used for *free-space electromagnetically propagated signals* and includes TV, etc.

[3] Pulse Amplitude Modulation (PAM) and Pulse Width Modulation (PWM) are other forms of pulse modulation

5.7 Time-division Multiplexing

One great advantage of Pulse-code Modulation (PCM) is the ease with which a number of signals can use the same physical channel, a process known as *Multiplexing*. In **Fig 5.6** we see an example of **time-division multiplexing (TDM)**[4], in which the digitised and binary coded samples from two inputs are interleaved to give ***frames***. Each frame is divided into a number of ***slots***, and the samples from each input are allocated to the same slot number in each frame.

To maintain synchronisation between input and output of the channel so that the output can be reconstructed in real-time, the length of a frame must be equal to the sampling period $\mathbf{1/f_s}$, and the length of a slot will depend on the number of signals that are multiplexed together.

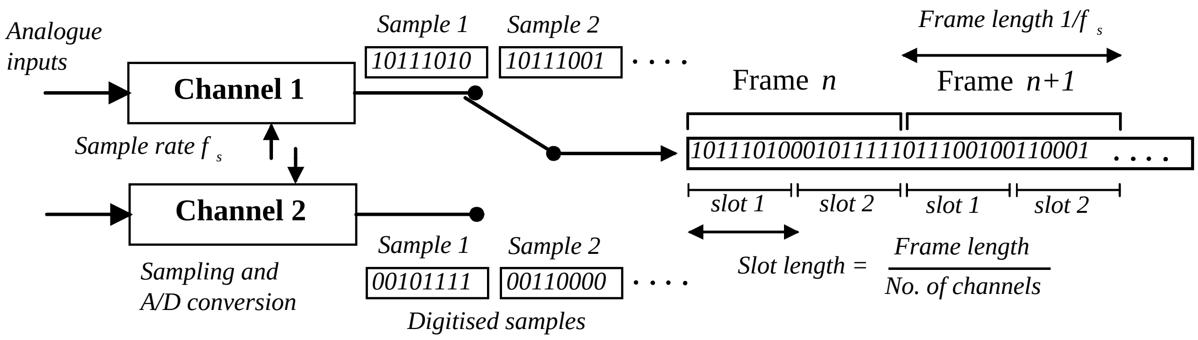

Fig 5.6 2-channel time-division multiplexing block schematic diagram

In a practical system, one or more channels will be used for signalling and synchronising purposes so that the ***de-multiplexer*** at the destination can keep in step with the frames and identify the slots.

5.7.1 The 32-channel PCM Transmission system

The 32-channel digital telephone system multiplexes 30 speech signals plus two control channels for signalling and synchronising. It has the following parameters:

- Signal bandwidth 3.4 kHz
- Sampling rate 8 kHz

  Hence frame length 125 μs
- Sample size 8 bits/sample

  Hence bit rate from each signal 64 kbit/s
- 32 channels

  Hence each time slot 3.906 μs

  Overall data rate 2.048 Mbit/s

If the time slots are shortened to a fraction of 3.906 μs, then a number of the basic “2 Mbit/s” bit streams or “Primary Multiplex Groups” can be time-division multiplexed together in a ***TDM heirachy***. (**Fig 5.7**)

For example, four of the 2 Mbit/s streams each with 32 channels gives a data rate of 8.192 Mbit/s (which is increased to 8.448Mbit/s with extra signalling bits) and contains 128 basic PCM channels. Four of these signals are multiplexed to give a 34.368 Mbit/s stream (512 channels), another 4:1 multiplexing produces a 139.264 Mbit/s stream with 2048 channels, and so on up to a multiplex of 32768 channels with an overall data rate of 2.48832 Gbit/s.

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[4] There is also Frequency-division multiplexing and Space-division multiplexing.

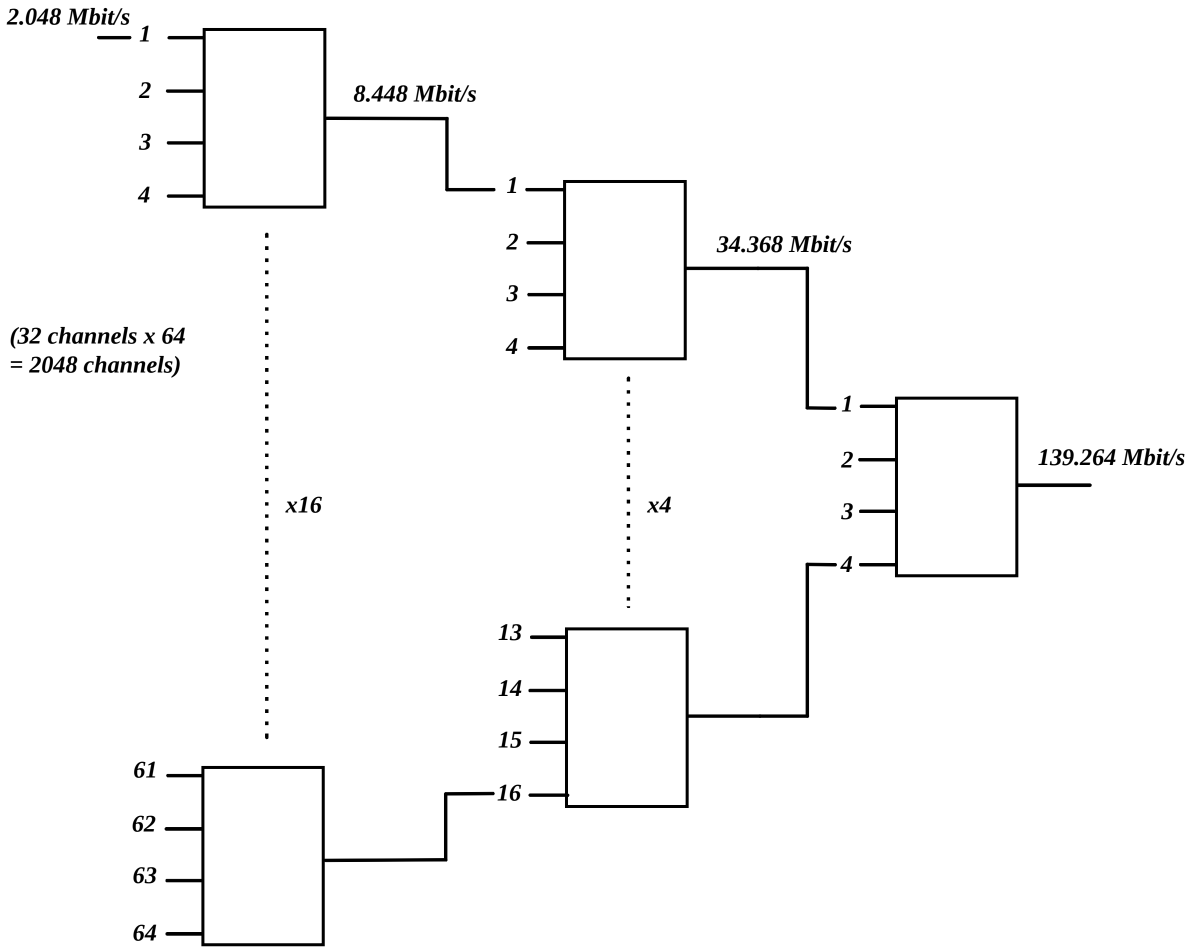

Fig 5.7 **A TDM heirachy using digital multiplexers**

Appendix A1 Spectrum of the sampling pulse