



Systems and Information Theory

Tutorial Assignment (PHE)

A Mars space probe takes colour stereo pictures at the rate of 1 per second which are relayed back to Earth by a radio communication channel. The stereo camera produces separate Left- and a Right-image video signals, each of which consists of 525 lines, and each line has 700 picture elements (*pixels*). Both stereo image signals contain two colour components, each of which can discriminate between 16 different colour hues, and a brightness component with 64 levels of brightness.

- a) Calculate the channel capacity required to transmit the video signal. (5)
- b) With reference to the Hartley Shannon Law equation

$$C = B \log_2(1 + S/N)$$

- (i) Explain why the quantity S/C could be called the *Energy per bit* (E_b). (2)

- (ii) Now show that $E_b/\eta = \frac{2^{C/B} - 1}{C/B}$ (where η is the *power spectral density* of white noise). (3)

- (iii) Plot E_b/η against C/B as C/B varies from 0.1 to 15 (use logarithmic scales)*. (5)

- (iv) Show from the graph that E_b/η approaches a limiting value as the channel bandwidth increases without limit and estimate this value. Show that this means that a power-limited channel has a maximum capacity no matter how large the bandwidth. (3)

- (v) Confirm this result from the Hartley-Shannon Law equation by deriving an expression for the maximum capacity as $B \rightarrow \infty$ in a white noise channel. Use the fact that $\lim_{x \rightarrow 0} \ln(1 + x)^{1/x} = 1$ (3)

- c) If the transmitter power is 20 W, the channel has a total of 180 dB attenuation and the noise is white with a power spectral density (η) of 1.0×10^{-24} W/Hz, show that the channel has sufficient capacity to transmit the signal, and find the minimum bandwidth necessary. (4)

(Total 25)

*A spreadsheet and its charting facility are convenient for this.