Systems and Information Theory - Tutorial PHE-1

Information Theory 1

Q1 It is discovered that Martians have an alphabet of 57 symbols. How many bits are required to code Martian letters into binary?

(6 bits)

Q2 a) What is the theoretical capacity of a channel with a bandwidth of 9 kHz and a signal-to-noise ratio (SNR) of 20dB?

b) What is the minimum theoretical bandwidth required for a broadband internet connection of 512kbit/s if the SNR is 10dB?

(60kbit/s, 148kHz)

Q3 A discrete channel is to convey information at the rate of 9 000 bit/s.

Determine the minimum number of symbol levels and SNR (in dB) required in theory, if the bandwidth of the channel is

a) 1 kHz b) 5 kHz

(32 levels, 27.1dB)
(2 levels, 3.9dB)

Q4 A modem operates at a bandwidth efficiency of 6 bit/s/Hz using 1024-level symbols over a channel of bandwidth 4 kHz.

a) How many bits are encoded in each symbol?

b) What is the baud rate?

c) How long would it take to transmit a message of 100 kBytes (1 Byte = 8 bits).

d) If the efficiency dropped to 4 bit/s/Hz because of noise on the channel, how long would transmission now take?

e) How many symbol levels would be required to transmit the message in half the time, assuming the baud rate remained constant?

(10 bits/symbol, 2400 baud)
(33.3s, 50s, 1048576 levels)

Q5 Starting with Eqn 8 , put $\mathbf{S_T}/\eta\mathbf{B_S} = \mathbf{y}$ and solve for **y.**

Now convert **y** into dB and plot the curve of $\mathbf{y_{dB}}$ against **b** for $(\mathbf{S}/\mathbf{N})_\mathbf{R}$ = 50 dB.

Hence estimate the minimum power required at the receiver input if the noise power spectral density is $7\text{x}10^{-22}$ W/Hz and the signal bandwidth is 500 kHz.

($11\ \text{x}10^{-15}$ W)

MathsAid - powers and logarithms:

1 *Manipulation of powers*

(i) $a^m.a^n = a^{m+n}$ (ii) $a^m/a^n = a^{m-n}$ (iii) $a^{mn} = (a^m)^n$ (iv) $a^{m/n} = {}^n\sqrt{a^m}$ (the n'th root of a^m)

2 *Definition of a logarithm*

If $x = a^n$ then, by definition, $n = \log_a x$

Note: (i) if x = a then $n = \log_a a = 1$

(i) if x = 1 then n = 0.

(ii) if x < 1 then n is negative

(iii) the log of a negative number is not a real.

3 *Changing the base of a logarithm*

$\log_a x = \log_b x/\log_b a$

4 *Use of logarithms*

If $n = \log_a x$ and $m = \log_a y$

then $x.y = a^n.a^m$

$= a^{n+m}$

and (i) $\log_a x.y = \log_a x + \log_b y$

Similarly (ii) $\log_a x/y = \log_a x - \log_b y$

(iii) $\log_a x^y = y\log_a x$ *from 1(iii)*

and (iv) $\log_a {}^y\sqrt{x} = (1/y)\log_a x$ *from 1(iv)*

5 *Logarithmic scales on graphs*

If you have a function like $y = ka^{mx}$

Take logs of both sides:

$\log_a y = \log_a k + m \log_a x$

or $Y = mX + K$

WhereY, X and K are the respective logarithms of y, x and k.

You now have the equation of a straight line which is easier to draw than the original function.

6 *Log-plotting of decibels:*

For the general low-pass response of a CR circuit,

$G_{dB} = -10 \log_{10} \{1 + (\omega CR)^2\}$

and if $\omega CR >> 1$, $G_{dB} \approx -20 \log_{10} \omega CR$

Putting $x = \log_{10} \omega CR$ we have

$G_{dB} \approx -20x$ which is a straight line of slope -20 dB per unit of x

Note that the straight line passes through $G_{dB} = 0$, x = 0 (or $\omega CR = 1$)

Now $(x + 1) = \log_{10} \omega CR + \log_{10} 10$

$= \log_{10} 10(\omega CR)$

so that a unit increase in x gives a 10 times change in ω - a **decade** and the slope is -20 dB per decade.

This is why frequency responses are plotted with gain in dB and the frequency axis in decades