

# Wavefront Coding for Athermalization of Infrared Imaging Systems

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## ABSTRACT

Wavefront coding involves the insertion of an asymmetric refractive mask close to the pupil plane of an imaging system so as to encode the image with a specific point spread function that, when combined with decoding of the recorded image, can enable greatly reduced sensitivity to imaging aberrations. The application of wavefront coding has potential in the fields of microscopy, where increased instantaneous depth of field is advantageous and in thermal imaging where it can enable the use of simple, low-cost, light-weight lens systems. It has been previously shown that wavefront coding can alleviate optical aberrations and extend the depth of field of incoherent imaging systems whilst maintaining diffraction-limited resolution. It is particularly useful in controlling thermally induced defocus aberrations in infrared imaging systems. These improvements in performance are subject to a range of constraints including the difficulty in manufacturing an asymmetrical phase mask and significant noise amplification in the digitally restored image. We describe the relation between the optical path difference (OPD) introduced by the phase mask and the magnitude of noise amplification in the restored image. In particular there is a trade between the increased tolerance to optical aberrations and reduced signal-to-noise ratio in the recovered image. We present numerical and experimental studies based on noise amplification with the specific consideration of a simple refractive infrared imaging system operated in an ambient temperature varying from 0°C to +50 C. These results are used to delineate the design and application envelope for which infrared imaging can benefit from wavefront coding.

Keywords: Wavefront coding, depth of field, defocus, athermalization, noise amplification.

## 1. INTRODUCTION

The conventional concept of imaging relies on the design of lenses that ensure rays diverging from a particular object point will intersect again at a corresponding point in the single image plane, building the image point by point. The brightness at any image location is proportional to the brightness of the corresponding object point, so that the image is a distribution of flux that is geometrically similar to that of the object.

A novel technique, known as wavefront coding, has been developed to increase the degrees of freedom or system trade-space<sup>1, 2, 3</sup>. Phase masks are designed to alter or code the transmitted incoherent wavefront in such a way that the point-spread function (PSF) is practically constant across a region near the focal plane, so the post-processed image can be accurately decoded by digital signal processing for a large range of defocus, as shown in Figure 1.

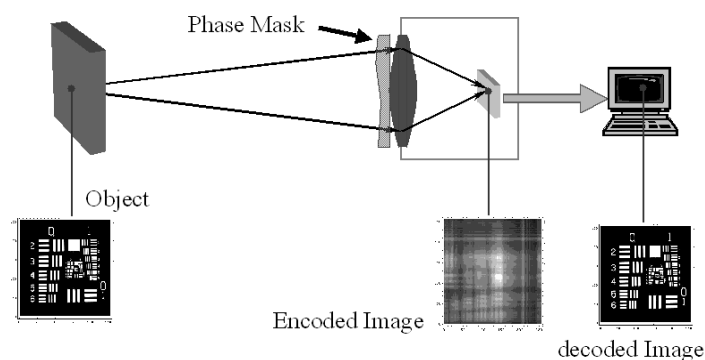


Figure 1. Principle of a wavefront coded imaging system.

By jointly optimising the optical coding and digital decoding, it is possible to achieve tolerance to defocus-related aberrations (temperature-induced defocus, field curvature, chromatic aberration) that could not be attained by traditional imaging systems. Furthermore, we have previously shown that wavefront coding enables reductions in sensitivity to third order aberrations (spherical, astigmatism, coma aberrations) whilst retaining high optical throughput<sup>4</sup>. The enhancement of the performance achieved by this wavefront coding technique is accompanied by a reduction in the signal-to-noise ratio of the restored image. The noise accompanying detection is amplified by the filtering process and there is hence a requirement for a trade-off between aberration tolerance and noise amplification in the restored image.

Wavefront coding promises to be particularly appropriate in thermal imaging, where there is an increase demand for low-cost LWIR systems. IR optical systems have predominantly been made with germanium lenses due to its high refractive index and absence of a need for achromatization (low dispersion due to high Abbe number). However, germanium is expensive and very sensitive to temperature changes producing significant focus shift with a few degrees of temperature change. The mechanical and optical techniques that athermalize or compensate for these thermal variations increase the complexity, weight and cost of IR optical systems.

The focus-invariant feature of wavefront coding jointly with digital decoding offers the potential to implement athermal and achromatic, diffraction-limited IR imaging using simple and low-cost optics. In this paper, we explore the use of wavefront coding as an athermalization technique in a practical optical system in which no thermal compensation has been included.

In the next section, a brief review of the theory of wavefront coding and experimental validation of this technique is given. In section 3, a practical implementation of wavefront coding as an athermalization technique in a real infrared lens design is investigated. Relationship between noise amplification and extended usable temperature range is presented.

## 2. CONCEPT OF WAVEFRONT CODING

Traditionally, aberrations are alleviated by introducing either optical modulation with amplitude or with phase filters, or by digitally processing the recorded image separately. Dowski and Cathey<sup>1</sup> proposed a new hybrid optical/digital configuration where an aspherical phase filter is used to produce an encoded image that is insensitive to defocus and some residual aberrations. The encoded image can then be accurately restored computationally. Various methods have been used for synthesizing the design of wavefront coding phase mask, for both square and circular apertures. Initially, the design of the phase mask was carried out in the frequency domain by the use of the ambiguity function which indicated that the ideal mask for an extended depth of field has a linear separable cubic form. In recent years, wavefront coding has been extended to include more general phase functions; phase masks have been developed in the spatial domain in which the point spread function (PSF), Strehl ratio, information metrics are solved to be invariant to defocus<sup>5,6</sup>. In this paper, the analysis of wavefront coding is restricted to the use of the cubic phase mask. This phase filter is designed from the evaluation of the ambiguity function of an incoherent imaging system by using the stationary phase approximation. The derived filter has a rectangularly separable anti-symmetric phase shape and is given by

$$P(x, y) = \text{Exp} [i \alpha (x^3 + y^3)] \quad (1)$$

where  $|x| < 1$ ,  $|y| < 1$  are normalised co-ordinates and  $\alpha$  is a real variable that controls the peak-to-valley magnitude of the optical path difference introduced. The strength of the phase mask  $\alpha$  sets the maximum phase deviation which yields a specific focus invariant region. A comparison of PSFs produced with and without the cubic phase mask is shown in Figure 2. The PSFs obtained with the phase mask are extended in comparison to the compact PSFs observed with a conventional lens and are practically invariant in the region near the focal plane. Because the cubic PSF are extended, the image obtained appears blurred. As expected, the PSFs obtained with the conventional system change significantly as the image plane is translated along the optical axis.

In the frequency domain, the wavefront coded MTF decreases relative to the diffraction-limited case as the spatial frequencies increase. The defocus invariance, and thus the MTF suppression, is strongly dependent on the strength or

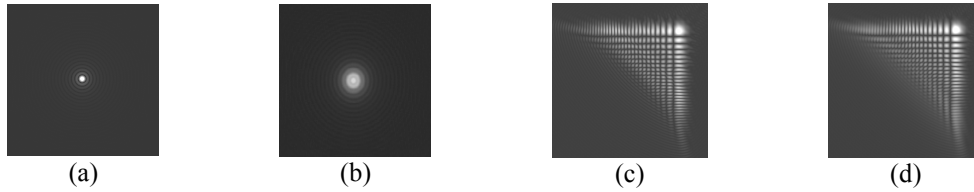


Figure 2. Simulated images of conventional point spread function in focus (a), with  $2\lambda$  of defocus (b), and cubic phase mask point spread functions in focus (c) and  $2\lambda$  of defocus.

peak-to-valley distance  $\alpha$  of the cubic phase function. Effective mitigation of severe defocus will require a phase mask with large phase-retardation  $\alpha$ . The magnitudes of an in-focus and out-of-focus MTF for a high-quality lens are shown in 3(a). In this case, even for small values of defocus, the MTFs go through numerous regions of zeros, where there is a phase shift, giving a contrast reversal in the image.

For comparison, Figure 3(b) shows the MTFs for the same lens with a cubic phase mask added in the aperture stop. Note that the MTFs are similar for any defocus, which implies that the imaging system will have an extended depth of field. Increasing the modulation coefficient  $\alpha$  results in lower values of the MTF compared with the conventional in-focus MTF. Furthermore, the coded MTFs for the defocus cases do not exhibit zeros. This means that the signal processing, which is needed to restore the MTF to that of the in focus conventional system, can be done without loss of information. During the deconvolution process a reduction in the signal-to-noise ratio (SNR) is expected. This noise amplification sets a practical limit to the system performance and depends substantially on the appropriate design of the phase filter. This disadvantage is in addition, to the difficulties that arise in manufacturing an aspherical phase filter such as a cubic mask, although in many cases an acceptable trade-off of noise amplification against aberration tolerance can be achieved.

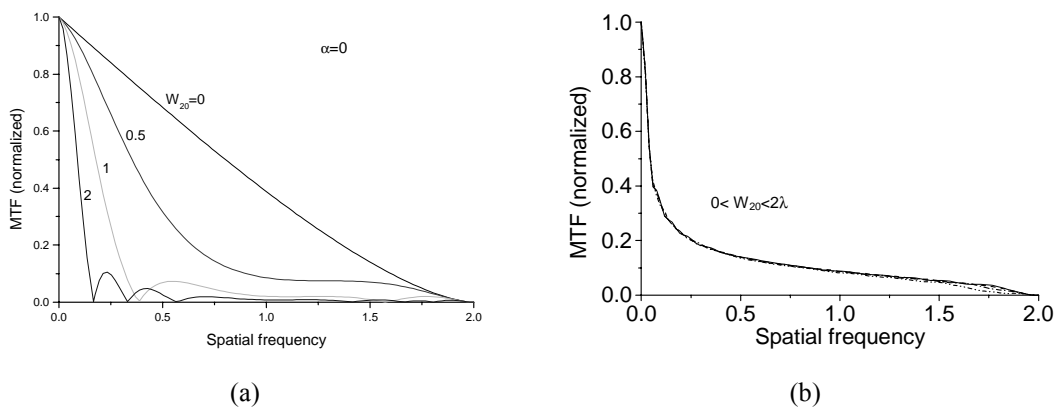


Figure 3. (a) One dimensional conventional system MTF for four focus positions ( $W_{20}=0\lambda, 0.5\lambda, 1\lambda, 2\lambda$ ), (b) MTF from a focus-invariant system for the same values of defocus used in (a) and with cubic phase mask.

### 1.1. Experimental verification of and extended depth of field optical system.

Experimental investigation in visible light was conducted using an etched cubic phase mask. Experiments with a visible band system enables risk reduced appraisal of wavefront coding prior to a more costly implementation in the far infrared. In the visible spectrum, we can readily carry out a series of experiments to evaluate the artefacts in the digitally processed image, the effect of detector sampling, the signal-to-noise ratio along various directions of the restored image (due to the cubic nature of the phase mask), the practical range at which the wavefront coded PSF remains invariant, etc.

The effect of defocus on the conventional imaging system with F/10, is shown by the experimental results shown in the right column of Figure 4. The best focus position corresponding to a value of  $W_{20}=0$  results in the sharply focused image

of Figure 4(a). Moderate defocus corresponding to a defocus value  $W_{20}=0.25\lambda$  results in the blurred image of Figure 4(b). Finally, the severely defocused images Figure 4(c) through to Figure 4(f) correspond to defocus values  $W_{20}=1\lambda$ ,  $1.5\lambda$  and  $2.5\lambda$  respectively.

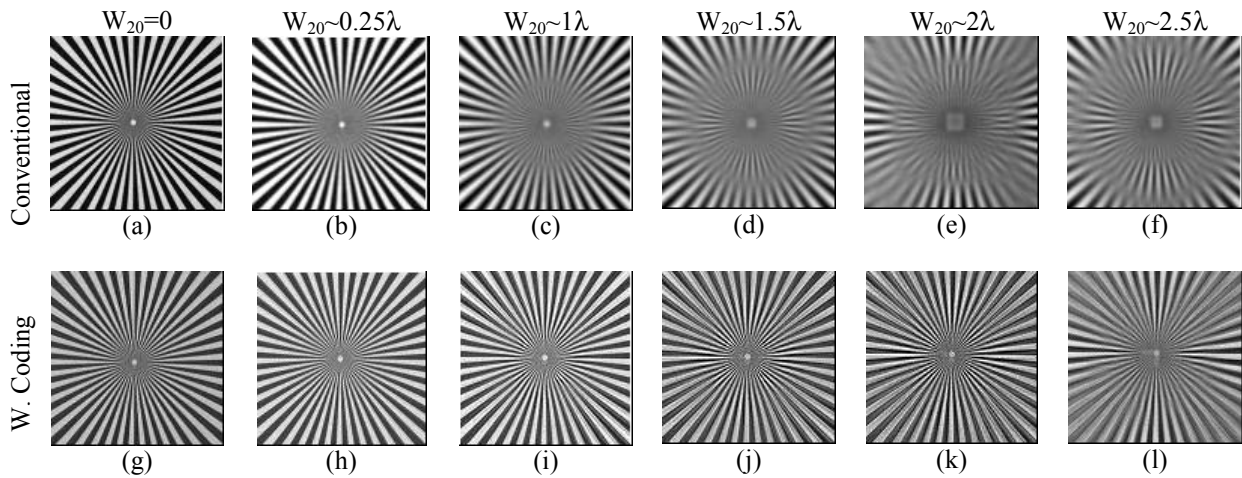


Figure 4. Top row column (a)-(f): images produced by a clear aperture imaging system for different degrees of defocus. Lower row (g)-(l): digitized filtered cubic phase mask images using a Wiener filter.

Note that across these images of the target one can observe contrast reversal because of the defocus introduced by the conventional optical system. The severely defocused images exhibit substantial loss of information about the original object. When the cubic phase mask is inserted at the aperture stop, it produces a considerable difference in the image formation performance, compare to the previous conventional clear aperture optical system. The intermediate coded images produced at the detector were highly distorted, but were restored in software using a Wiener filter. The upper row in Figure 4 shows the resultant digitally processed images corresponding to the same defocus values given above. Most of the images have a close resemblance to the well-focused image of Figure 4(a). The significant degree of similarity between the wavefront coded images over a wide range of defocus indicates that the optical system is highly focus invariant. Note that, in contrast to the conventional system, there are no phase changes across the images. Each of the images Figure 4(g) through to Figure 4(l) was decoded using a single filter constructed from the same in-focus point spread function.

Similarity of results tells us that artefacts associated with practical implementation are not a major problem enabling low-risk design of TI systems.

Finally, the imperfections in the reconstructed images Figure 4(k) and Figure 4(l) are caused mainly by the extreme values of uncorrected defocus and the design of the cubic phase mask, which along one diagonal slice the OPD is constant whilst the other diagonal of the OPD is a cubic. There is a slow change of MTF with defocus and this change is faster for spatial frequencies oriented across the diagonals than along the x-y axes; this results in a greater loss of contrast for diagonal frequencies. This effect can be mitigated using alternate wavefront coding masks using different symmetry characteristics.

These images are useful for qualitative comparison purpose and for determining the amount of extended depth of field.

### 3. ATHERMALIZATION OF AN INFRARED SYSTEM WITH WAVEFRONT CODING

Focus shift with temperature presents a significant problem in thermal imagers<sup>7</sup>. Conventional infrared optical materials exhibit large variations in index of refraction with temperature. For example, the change of refraction with temperature for Germanium is over 100 times that of common optical glasses of the visible spectrum. Athermalization is the correction of this effect of focus shift with temperature. There are many mechanical and optical techniques available to accomplish athermalization. These include using specific combinations of optical materials, using temperature sensors to

actuate motors which realign the optical elements, using optical housing materials that change with temperature such as to compensate the changes in optical properties, using diffractive optics<sup>8, 9, 10</sup>. Wavefront coding may be used in combination with these common athermalization methods to reduce the amount of athermalization needed in the thermal imager. Moreover, wavefront coding can be employed to increase the working temperature range of the imager given a fixed amount of athermalization<sup>2</sup>.

As a result of the variations in refractive index and radii of curvature, the thermal misfocus over a temperature range of  $\Delta T$  for a lens element with focal length  $f$  is

$$\Delta f = -\gamma f \Delta T \quad (2)$$

where  $\gamma$  is analogous to a “focal length expansion coefficient”, namely the fractional change in focal length per degree of temperature change. The thermal constant  $\gamma$  for the material is given by

$$\gamma = \frac{dn/dT}{n-1} - \alpha \quad (3)$$

with  $n$  the refractive index of the material and  $\alpha$  the expansion coefficient. The focal shift  $\Delta f$  due to temperature variation  $\Delta T$  gives rise to a wave aberration

$$W_{20} = \frac{\Delta f}{2} \left( \frac{1}{2F\#} \right)^2 \quad (4)$$

where  $F\#$  is the f-number (ratio of focal length to entrance aperture). For instance, a simple spherical infrared imaging lens composed of Germanium with  $f=20\text{mm}$  ( $\sim F/1$ ) designed to operate at  $20^\circ\text{C}$ , shifts its focal length by nearly  $\pm 80\mu\text{m}$  when the temperature varies from  $-10^\circ\text{C}$  to  $+50^\circ\text{C}$ . The corresponding defocus is  $W_{20} \sim 1\lambda$ . Although this traditional infrared system is diffraction-limited at  $20^\circ\text{C}$ , this performance cannot be maintained over a large change in temperature.

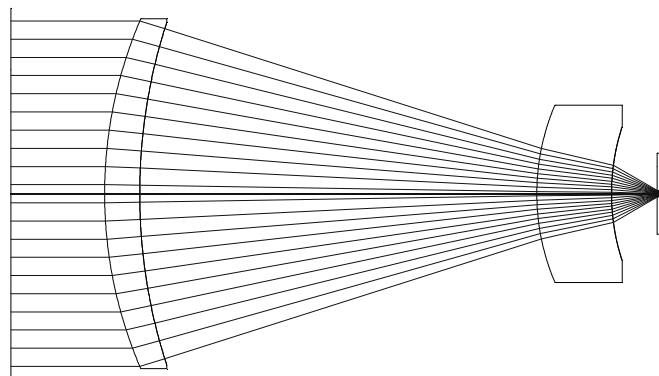


Figure 5. Lens layout of infrared imaging system, effective focal length  $f=75$  mm,  $F/1$ , half field of view of  $6^\circ$ .

An example of the potential of wavefront coding to extend the usable temperature range of a thermal imaging system with no athermalization is presented. The design goals for this imager are an  $F/1$  optical system with an effective focal length of  $75\text{mm}$  and  $\pm 6^\circ$  half field of view as shown in Figure 5. The operating wavelengths are  $8\text{--}12\ \mu\text{m}$  and the system is designed to be diffraction limited at  $+17^\circ\text{C}$ .

The lens is based on the Petzval configuration, which includes a germanium aspherical hybrid front element and a thick spherical germanium meniscus element. The detector window is also made of Germanium. Because all the elements are made of germanium, the performance of the imaging system is extremely sensitive to temperature variation. Figure 6 shows the MTF of the system as a function of temperature. With a change of temperature of about  $\pm 7^\circ\text{C}$ , the image quality becomes degraded; in this case, the value of the defocus parameter is  $W_{20} \sim 0.5\lambda$ . With temperature changes of more than  $\pm 10^\circ\text{C}$  the system becomes severely defocused and all the MTFs contain zeros. It is evident that this generic system requires thermal stabilization in order to operate even over a modest range of temperatures.

When a cubic phase mask ( $\alpha=8\pi$ ) is inserted in the front of the infrared lens, the MTFs for different temperature values (i.e. different degrees of defocus) remain essentially constant and very similar to each other as shown in Figure 6(b). Furthermore, these MTFs have no zero values, in contrast with those from the traditional systems in Figure 6(a). This allows application of a simple inverse digital filter and the system can be restored to an almost diffraction-limited performance over a  $50^\circ\text{C}$  temperature range. Note that the applied digital filter is the same for all the MTFs. Certainly, another alternative would be to use post-detection signal processing in the conventional IR only with a temperature sensor inserted<sup>2</sup>. The sensor would inform the determination of the appropriate PSF to be employed as the inverse filter so the image could be restored for all temperatures. However, the corresponding MTFs would contain zeros which would yield loss of information in the final image.

As mention above, the cubic element reduces the magnitude of the resulting MTFs in comparison with the diffraction-limited performance of the IR lens. This drop in the signal height represents significant noise amplification in the digitally processed image. The suppression in the MTFs is determined by the “strength”  $\alpha$  of the cubic phase mask which determines also, the degree of athermalization. Acceptable athermalization performance can be achieve according to the noise amplification and temperature range trade-off set by the requirements of the application as we will see in the next section.

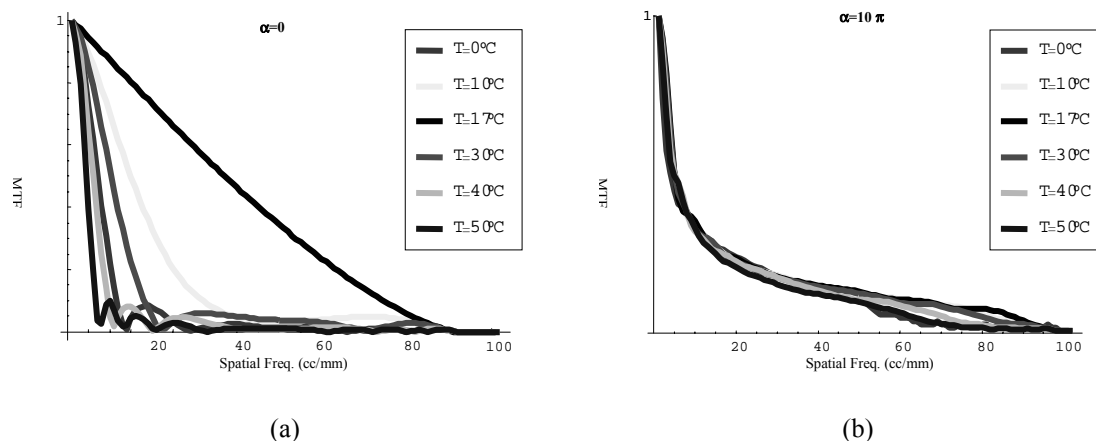


Figure 6. (a) MTF of the proposed real infrared lens as a function of temperature. The lens is design to be diffraction limited at  $+17^\circ\text{C}$ , (b) MTF of the athermalized infrared lens by means of wavefront coding.

The magnitude of noise amplification is determined by the amplification introduced by deconvolution operations using simple inverse filters and Wiener filters to restore wavefront coded images. This simple linear restoration allows us to derive straightforward merit functions to explore the performance of wavefront coding IR system. More complex filters with computationally demanding algorithms, such as iterative and non-linear methods, give better restoration fidelity but do not readily facilitate a merit function for a noise amplification analysis<sup>11</sup>. The noise amplification as a function of athermalization was calculated for the given wavefront coded IR system after deconvolution, the MTF of the hybrid optical/digital system matched the conventional diffraction-limited MTF.

Figure 7 shows plots corresponding to the noise amplification of different cubic phase masks as a function of operational temperature range after inverse filtering. Initially, the intermediate image recorded at the detector contains 1% of

additive white Gaussian noise. For each temperature, the image is restored using the PSF kernel which corresponds with that temperature. Thus, the restored image quality in terms of MTF is diffraction-limited (the image quality is kept constant along the temperature axis). When  $\alpha$  is zero, the mask is flat and of course the noise amplification factor is unity at the designed temperature. In this case, the performance of the system is reliable only over a few degrees until it attains infinite noise amplification at  $\pm 7^\circ\text{C}$ , where there is the first null in the MTF. As expected, the noise amplification increases with the value of  $\alpha$ . It can be observed that if we want to increase the temperature range in the IR system over  $\pm 30^\circ\text{C}$  it is necessary to set at least  $\alpha = 4\pi$ . The associated noise amplification would be of from 3 to 4. Further athermalization would require a larger  $\alpha$ .

Note that variation of noise amplification within a chosen temperature range for a given phase mask, indicates that focus-invariance or complete athermalization have not been achieved as the phase mask cannot compensate completely for defocus associated with that temperature; the region of complete athermalization is indicated by the region for which amplification remains constant with temperature. Therefore, depending on the requirement of the application, we may deduce from Figure 7 the optimum phase strength that achieves the required athermalization and the corresponding noise amplification.

### Noise Amplification

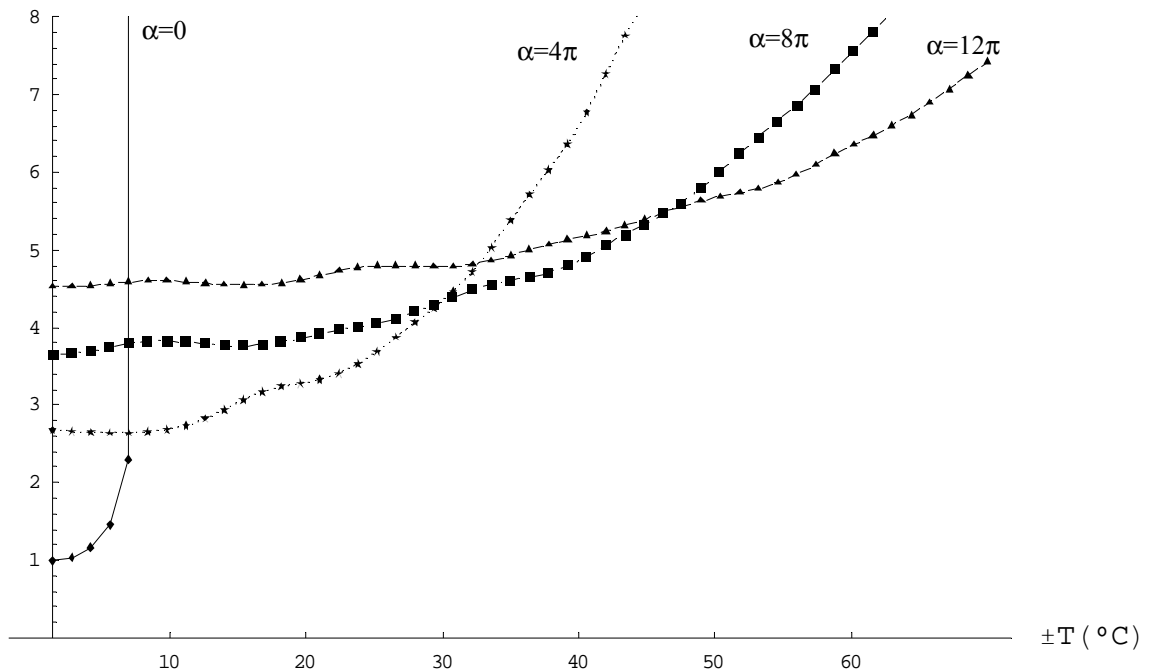


Figure 7. Simulated noise amplification as a function of temperature after inverse filtering for various values of  $\alpha$ .

The data in figure 7 represents the noise amplification effects when deconvolution is executed so as to always achieve diffraction-limited imaging. In practice it is more likely that a Wiener filter would be used to avoid excessive amplification of noise; the MTF in this case will exhibit nulls where the MTF of the signal recorded at the detector is very low. In this case, unlike the inverse filter situation, the noise amplification curves start to fall when there still exist nulls in the restored MTF which will not contribute to noise amplification due to the very nature of the Wiener filter. This filter cancels any noise and signal contribution from and around the points of zero value in the MTF. The size of the suppressed area around the nulls is controlled by the constant (in the denominator of the Wiener filter) chosen to optimize the estimate. In Figure 8, the noise amplification curve for  $\alpha = 4\pi$  starts falling at  $30^\circ\text{C}$  as wavefront coding is unable to correct larger amount of defocus. In conclusion, the reduction in noise amplification is accompanied by a loss in signal. It is therefore required to increase  $\alpha$  to maintain focus invariance across at the expense of noise amplification.

Analysis showed in Figure 7 enables to extract the amount of modulation  $\alpha$  needed in the cubic phase mask so as to athermalize the IR system, this occurs when the noise amplification is constant over a temperature range. In our case, the corresponding modulation for our goal temperature range of 0°C to 50°C is  $\alpha=12\pi$ . This value is now taken into Figure 8 to determine the noise amplification if a Wiener filter was to be used, which gives, for our example, a noise amplification of less than 3.5.

Noise Amplification

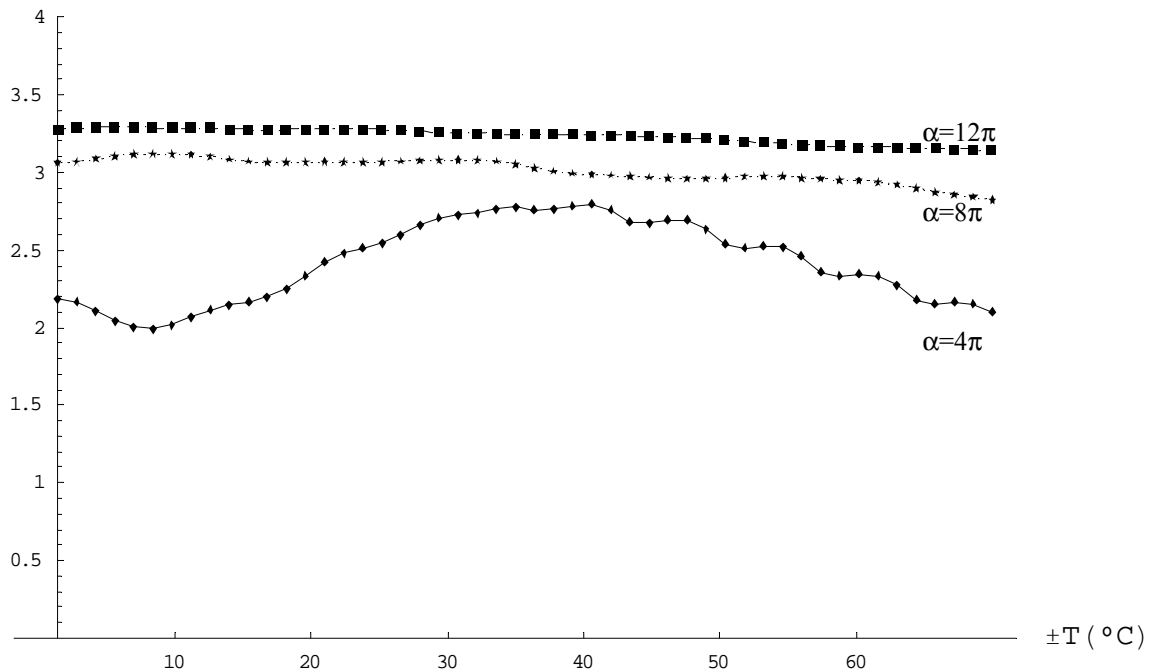


Figure 8. Simulated noise amplification as a function of temperature after Wiener filtering for various  $\alpha$ . The noise amplification decreases as the phase mask cannot compensate for large amounts of defocus.

#### 4. CONCLUSION

A review of the concept of wavefront coding together with experimental validation of a hybrid optical/digital imaging system with extended depth of field were presented. It was shown that pupil plane encoding offers a low-cost and light-weight alternative to existing athermalization techniques. This extended performance is balanced by noise amplification in the restored image. As an example, the noise amplification of a wavefront coded IR imaging system was modelled for different usable temperature ranges. A noise amplification factor of approximately 4.5 was obtained when the IR imager is athermalized from 0°C to 50°C after inverse filtering and a factor less 3.5 in comparison to the use of a Wiener filter. The appropriate cubic phase mask to totally athermalize the system over that temperature range requires  $\alpha=12\pi$ . In the future, improvement in detector sensitivity will reduce system noise in uncooled detectors and it will provide greater scope for the use of wavefront coding in low-cost thermal imager whilst maintaining current performance levels.

#### ACKNOWLEDGEMENTS

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#### REFERENCES

1. E. Dowski and T. W. Cathey, "Extended depth of field through wavefront coding," *Appl. Opt.* **34**, 1859-1866 (1995).
2. W. T. Cathey and E. R. Dowski, "New paradigm for imaging systems," *Appl. Opt.* **41**, 6080-6090 (2002).
3. K. Kubala, E. R. Dowski, and W. T. Cathey, "Reducing complexity in computational imaging systems," *Opt. Express* **11**, 2102-2108 (2003).
4. S. Mezouari, G. Muyo and A. R. Harvey, "Amplitude and phase filters for mitigation of defocus and third-order aberrations" *Proc SPIE* 4768, 21-31 (2003).
5. S. S. Sherif, E. R. Dowski, and W. T. Cathey, "Extended depth of field in hybrid imaging systems: circular aperture," *J. Mod. Optics*, 2004, 1-19.
6. S. Prasad, T. Torgersen, V. P. Pauca, R. Plemmons, J. van der Gracht, "Engineering the Pupil Phase to Improve Image Quality," in *Proceedings of the SPIE, Vol. 5108 Visual Information Processing XII*, edited by Z. Rahman, R. Schowengrdt, and S. Reichenbach (SPIE, Wellingham, WA, 2003), pp. 1-12.
7. A. Mann, *Infrared Optics and Zoom Lenses*, SPIE press, Bellingham, WA, 1999.
8. T.H. Jamieson, "Thermal effects in optical systems," *Opt. Eng.* **20**, 156-160 (1981).
9. D.S. Grey, "Athermalization of optical systems," *J.Opt.Soc.Am.* **38**, 542-546 (1981).
10. G.P.Berhmann and J.P.Bowen, "Influence of temperature on diffractive lens performance," *Appl. Opt.* **32**, 2483-2489 (1993).
11. J. van der Gracht, J. Nagy, P.Pauca, and R. Plemmons, "Iterative restoration of wavefront coded imager for focus invariance," in *OSA Trends in Optics and Photonics (TOPS), Integrated Computational Imaging Systems*, OSA Technical Digest, Washington, D.C., 2001.