Band pass active aperture synthesis using spatial frequency heterodyning

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Abstract. In this aperture synthesis three snapshots are required to synthesis the aperture: one is obtained with simple illumination of the scene, the second when the scene is projected with cosine fringes and the third with the fringes quadrature phase shifted. The spatial frequency of the fringes is set to twice the cut off frequency of the imager. Due to spatial frequency heterodyning bands of scene spatial frequencies lying both above and below the fringe spatial frequency are heterodyned simultaneously into the low-pass band of the imager Computer simulation results show extension of cut off frequency of the imager to three times the cut off frequency without aperture synthesis.

1. Introduction

A conventional imager acts as a low-pass filter to the spatial frequencies of a scene and consequently images recorded by such an imager have restricted resolution. Several techniques have been proposed in the literature [1-13] to enhance this resolution when the dimensions of the imager are fixed and the images are recorded under monochromatic illumination.

Post-detection signal processing methods may be used to improve the spatial resolution, as determined by some heuristic resolution criterion (such as the Rayleigh criterion), by restoration of the pass-band filtered Fourier spectrum. These methods however enhance a discontinuity in the cutoff frequency in the spectra, resulting in strong Gibbs ringing in the restored image. Gibbs phenomena may be reduced by the use of a class of super-resolution algorithms that seek to infer the information beyond the cutoff frequency [2-4]. The performance of these algorithms depends on the signal-to-noise ratio of the images and on some a priori knowledge of the scene characteristics obtained using analytical or statistical methods [5-7] and the properties may vary from one scene to another.

There are several techniques that help to encode the higher spatial frequency content of the scene onto lower frequency regions, so that all the spatial-frequency information can pass through the limited bandwidth of the imaging system. This concept was first used in the Lukosz techniques, reported by Lukosz,[8] and Sun and Leith,[9] which employ an interferometric mechanism consisting of multiple moving diffraction gratings. These gratings, when placed in front of the imaging lens, produce coupling between the spatial and temporal frequencies, so that higher spatial frequencies are shifted onto lower frequency bands that are transmitted by the optical system. These shifted bands have different temporal frequencies and may be restored by the use of a conjugate interferometric system positioned behind the lens. Another similar technique for the bandwidth extension of IR detectors through spatial-frequency shifting by the use of a sampling grating in a number of predetermined positions has been reported [10].

In this paper a new scheme is proposed for extending the bandwidth of an optical imager. The principle of operation is analogous to that of the time-domain operation of a radio-frequency spectrum analyzer in which a high-frequency local oscillator is mixed with a high-frequency radio signal to be measured. The local oscillator frequency is adjusted such that the beat frequency produced between it and the signal is low enough to fall within the pass-band of a low-frequency spectrum analyzer. In our proposed technique, high spatial frequency cosine interference fringes illuminate the scene and heterodyne the scene spatial frequencies close to the fringe frequency to lower spatial frequencies that are transmitted by the low-pass filter characteristics of a conventional imager. Convolution of the measured heterodyned components with the fringe frequency shifts the scene components back to their frequency in the scene. Reconstruction of a high resolution image consists of a superposition of low-pass and band-pass heterodyned images in the spatial frequency domain to produce the Fourier transform of high resolution image which is inverse Fourier transformed to produce the image. The use of a fringe frequency equal to twice the cutoff frequency of the imager increases the effective cutoff by a factor of three. Additional high-frequency components can be added by increasing the fringe frequency further to enable images of very high angular resolution images to be constructed. Complications that arise include the fact that both upper and lower sidebands and the base-band image components occur simultaneously in the heterodyned image, in general the fringes are curved rather than straight and all the recorded images are accompanied by phase errors in the projected fringes and also by both systematic speckle noise and random noise.

A mathematical description of the process by which scene is constructed is presented in section 2. An example illustrating the technique is shown in section 3. The remaining sections include the effect of phase error, curved fringes, detector and speckle noise on the quality of reconstructed images.

2. Mathematical modelling in 1D

The concept presented here is valid both for coherent and incoherent imaging. Mathematical modeling for incoherent imaging is presented to establish the concept, which can be modified for coherent systems. The three snapshots of a scene g(x) for an incoherent diffraction limited imager are given by:

$$s'_b(x) = g(x) \otimes h'(x) \quad (1)$$

$$s'_{bc}(x) = g(x)(1+\cos(2\pi.f.x)) \otimes h'(x). \quad (2)$$

$$s'_{bs}(x) = g(x)(1+\sin(2\pi.f.x)) \otimes h'(x) \quad (3)$$

where f is the spatial frequency of the fringes, ⊗ stands for convolution $s'_b(x)$, $s'_{bc}(x)$ and $s'_{bs}(x)$ are the intensity images corresponding to three snapshots. h′(x) is the point spread function of the imager. Assuming that the transfer function $H'(k) = \Im[h'(x)]$ where $\Im$ is the Fourier transform operator is known. If the Fourier transform of the above equations are normalised with $H'(k)$, the above equations will take the following forms.

$$S_b(k) = G(k)H(k). \quad (4)$$

$$S_{bc}(k) = \left(\frac{1}{2}G(k+f) + G(k) + \frac{1}{2}G(k-f)\right).H(k) \quad (5)$$

$$S_{bs}(k) = \left(\frac{\mathrm{i}}{2}G(k+f) + G(k) - \frac{i}{2}G(k-f)\right).H(k) \quad (6)$$

where $S_j(k) = \frac{S'_j(k)}{H'(k)}H(k)$ and j stands for subscripts b, bc and bs. $S'_j(k) = \Im\lfloor s_j(x)\rfloor$ and $H(k)$ is unity in the pass band ($|\mathrm{k}| \le f/2$) and zero outside. The above transformation will help to visualize the concept more easily. The above equations give the following unknowns.

$$G(k+f).H(k) = (S_{bc}(k) - S_b(k)) - i.(S_{bs}(k) - S_b(k)) \quad (7)$$

$$G(k-f).H(k) = (S_{bc}(k) - S_b(k)) + i.(S_{bs}(k) - S_b(k)) \quad (8)$$

The left hand side of Eq.(7) represents upper side band of spatial frequencies which heterodyned into the low pass band of the imaging lens while the left hand side of Eq.(8) represents the lower side band of spatial frequencies which heterodyned into the low pass band of the imaging lens.
To assign the lower side band frequencies and the upper side band frequencies their actual positions in the Fourier spectra of the scene, it is necessary to de-convolve each contribution with the Fourier spectrum of cosine fringes i.e. frequency shift of bands by de-convolving.
De-convolution of left hand side of Eq.(7) with the Fourier transform of Cosine fringes gives

$$\begin{aligned} & G(k+f)H(k)\otimes\left(\frac{1}{2}\delta(k+f)+\delta(k)+\frac{1}{2}\delta(k-f)\right) \\ & =\frac{1}{2}G(k+2f)H(k+f)+G(k+f)H(k)+\frac{1}{2}G(k)H(k-f). \end{aligned} \tag{9}$$

Each term of Eq.(9) is separable and the third term is the desired term. De-convolution of left hand side of Eq.(8) with the Fourier transform of Cosine fringes gives

$$\begin{aligned} & G(k-f)H(k)\otimes\left(\frac{1}{2}\delta(k+f)+\delta(k)+\frac{1}{2}\delta(k-f)\right) \\ & =\frac{1}{2}G(k)H(k+f)+G(k-f)H(k)+\frac{1}{2}G(k-2f)H(k-f). \end{aligned} \tag{10}$$

Each term of Eq.(10) is separable and the first term is the desired term. Combining the right hand side of Eq.(4), third term of Eq.(9) and the first term of Eq.(10) to construct the extended Fourier transform $S_E(k)$ of the scene which is given by

$$S_E(k)=G(k)H_E(k) \tag{11}$$

$H_E(k)$ is the extended transfer function of the imaging lens and is related to H(k) by the following equation.

$$H_E(k)=H(k+f)+H(k)+H(k-f) \tag{12}$$

The inverse Fourier transform of Eq.(11) gives the extended band width version $s_e(x)$ of $s_b(x)$ and is given by

$$s_e(x)=g(x)\otimes h_e(x) \tag{13}$$

where $h_e(x)=\Im^{-1}[H_E(k)]$ is the point spread function of the synthesized aperture. $H_E(k)$ is unity for $|k|\leq 3f/2$ and is three times wider than $|k|\leq f/2$.

Comparison of $H_E(k)$ with H(k) implies a band width extension of three times. By increasing the spatial frequency of projected fringes, it is possible to extend the band width of the imaging system for a particular band of frequencies of interest. The above analysis shows an extension of three times but in principle, it is possible to extend the bandwidth almost indefinitely by selecting even integral values of f/2, one at a time. This can also help to restore a particular band of interest of scene spatial frequencies. The technique may be named Spatial Band-pass Telescopy (SBT).

3. Illustration of the technique

In this section we illustrate the application of the method in two dimensions. A set of equations similar to Eqs. (1)-(13) may be trivially derived for the two dimensional case but are not presented here. The illustration of the technique for 2-D images is presented in Fig. 1. The original scene of Lena of size 128 x 128 pixels is shown in Fig. 1(a) and its spectrum in Fig. 1(b). Fig. 1(c) shows a band limited version of Fig. 1(a) as seen by an imager with a bandwidth 1/3 that needed to produce an image in Fig. 1(a). The spectrum corresponding to Fig. 1(c) is shown in Fig. 1(d). Fig. 1(e) is an extension of bandwidth of the spectrum shown in Fig. 1(d) when the algorithm was applied in the horizontal direction. The application of the algorithm in the vertical direction yields an extended bandwidth spectrum in the vertical direction and is shown in Fig. 1(f). Extension of bandwidth of the spectrum in the left and right diagonal directions are shown in Figs. 1(g, h), respectively. The spectrum shown in Figs. 1(e, f, g, h) are combined to form full spectrum similar to the one shown in Fig. 1(b), the inverse FT of which is shown in Fig. 1(i). The image shown in Fig. 1(i) is a synthesized super-resolved image and the resolution of this image, for this particular illustration, is three times that of the band limited image shown in Fig. 1(c). The comparison of the image shown in Fig. 1(i) with that of the original image shown in Fig. 1(a) indicates that the technique works well for the 2-D images.

The result presented in Fig. 1 was simulated under the ideal conditions, which are (a) to project straight, high quality cosine and sine fringes onto the scene, (b) the central cosine fringes must lie at the centre of the field of view of the scene, (c) the sine fringes are accurately in quadrature phase with the cosine fringes, (d) the spatial frequency of these fringes are known accurately, (e) the cut off frequency of the imager is determined accurately, (f) the bandwidth of the imaging system is limited by the imager itself and not by the CCD, (g) speckle and white noise are negligible, (h) imager is a diffraction limited imager. In the next sections we discuss the qualitatively the impacts of deviations from these ideal conditions and consider under what conditions these impacts are negligible.

4. Deviation from ideal conditions: impact on image quality and possible solution

Here we qualitatively discuss the impact on the reconstructed images when system parameters deviate from the ideal conditions. The straight high quality cosine and sine fringes are required which are possible in a limited field of view. Simulation has shown that even if the fringes are hyperbolic a high quality image can be reconstructed provided that the distribution of the fringes at the scene is known. As obvious from Eq. (9) and (10) that the distribution of the fringes at the scene must be known in order to carry out the de-convolution process. If a third mutually coherent source, on either side, is located collinearly with the other two sources used to project fringes with the separation half that of the two sources, the imager would be able to see the fringes due to lower spatial frequency. If the phase of this source is scanned, the interferogram will move on to the other interferogram with twice the spatial frequency. When the visibility of the fringes attains maximum visibility the two sets of intergerograms exactly overlap. Under this condition, the distribution of the fringes with twice the spatial cut off frequency of the imager can be found. The third requirement that the sine fringes should be accurately in quadrature phase with the cosine fringes is possible and can be achieved [11]. We reconstructed a two beam fibre based interferometer and achieved phase accuracy better than 50 mrad which is quite adequate for this aperture synthesis technique. Further improvement in the phase accuracy can be achieved using amplitude and frequency stabilized laser, equalising the path length difference between the two fibre arms to within mm accuracy and with a better feed back control system. The spatial frequency of the fringes should be twice the spatial cut off frequency of the imager which can be set if the separation between the coherent sources is twice the dimensions of the imager along the corresponding imager axes. It is required that the speckle distribution should be the same during the three measurements which requires that the relative positions of sources, imager and scene should remain fixed during the measurements. Space, polarisation, time or frequency diversities are the possible options to minimise the speckles provided the requisite conditions are met [12]. Speckles can also be avoided if the temporal coherence of the sources is much shorter than the depth of the scene.

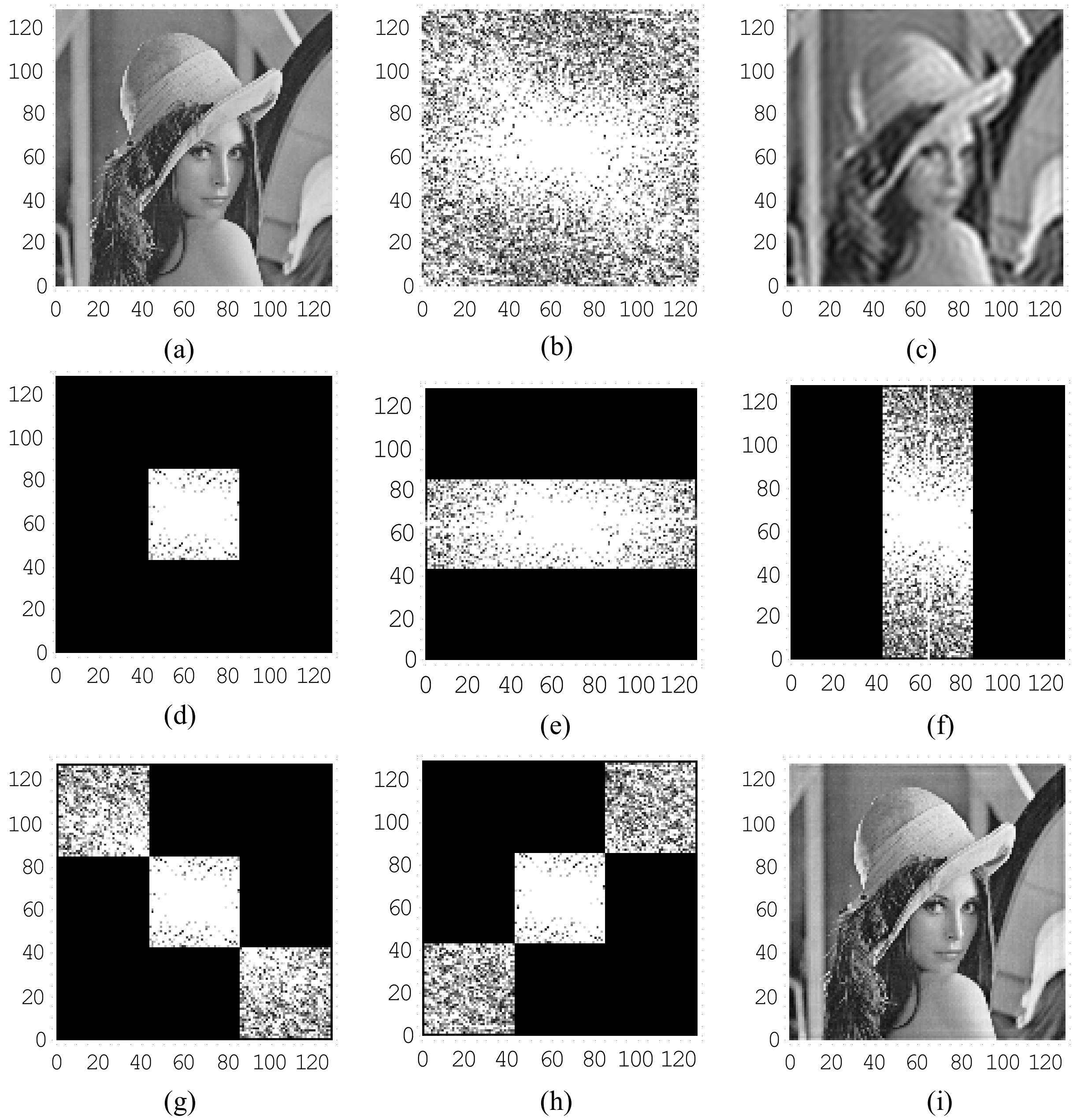

Fig. 1. Computer-simulated, 2-D example: (a) the original image, (b) the spectrum corresponding to (a), (c) image corresponding to (a) as seen by band limited imager, (d) the spectrum corresponding to (c), (e) bandwidth extension in the horizontal direction, (f) image corresponding to (e), (g) bandwidth extension in the vertical direction, (h) image corresponding to (g), (i) bandwidth extension in the left diagonal direction, (j) image corresponding to (i), (k) bandwidth extension in the right diagonal direction, (l) image corresponding to (k), (m) bandwidth extension in all directions, (n) image corresponding to (m).

5. Conclusion

This paper has described a super-resolution technique based on heterodyning of higher scene spatial-frequencies into the pass band of an imager for the case of coherent imaging. The illustrations for 2-D described above indicate an extension of bandwidth three times that of the pass-band but in principle it is possible to extend the bandwidth to indefinitely by repeating the technique for fringe spatial-frequencies equal to nf where n is an integer and f/2 is the cut-off frequency of the pass band of the imager. The technique is also applicable to any band of electromagnetic waves. Deviation from ideal requirements for the proposed technique may be due to experimental limitations, which have been discussed with computer-simulated results. The effects due to detector noise and speckle noise have also been incorporated in the computer-simulated results. It seems that the technique promises to present a worth solution for enhancement of angular resolution of an imager.

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