

Comment on ‘extended depth of field in hybrid imaging systems: circular aperture’

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A recent paper by Sherif *et al.* (S.S. Sherif, E.R. Dowski, W.T. Cathey, J. Mod. Opt. **51** 1191 (2004).) reported the derivation of an aspherical phase plate, which when placed at the exit pupil of a conventional imaging system and combined with digital processing of the recorded images, increases the depth of field by an order of magnitude. An error in the derivation of this phase plate has been identified, which makes the reported extension of depth-of-field sub-optimum rather than invalid. In this comment, an optimum phase plate is obtained and the relevant results are repeated.

1. Introduction

In a recent paper [1], Sherif *et al.* described a spatial-domain method to derive a pupil-plane phase plate, which extends the depth of field of an incoherent optical system with a circular aperture. By digitally post-processing the output of the detector, the depth of field was extended by an order of magnitude more than the Hopkins criterion.

A preliminary expression for the phase plate $f(r, \theta)$ in normalized polar coordinates (r, θ) was given by

$$f(r, \theta) = \vartheta(r, \theta) - \frac{\rho' r_{\max} r \cos(\theta - \phi')}{z_i}, \quad (1)$$

where $\vartheta(r, \theta)$ is an additional phase function, r_{\max} is the radius of the exit pupil, z_i is the image distance, ρ' and ϕ' are constants whose optimum values, for a given imaging system, were obtained by optimization using an optical design program. Sherif *et al.* demonstrated that the term $\vartheta(r, \theta)$ in equation (1) must satisfy the following expression, which can be considered as the condition for extended depth-of-field in optical systems with circular apertures:

$$\begin{aligned} & 3r\vartheta''_{rr}(\vartheta''_{\theta\theta})^2 - 3r\vartheta''_{\theta\theta}(\vartheta''_{r\theta})^2 - 3\vartheta'_r(\vartheta''_{\theta\theta})^2 + r\vartheta'_r\vartheta'''_{\theta\theta r}\vartheta''_{\theta\theta} \\ & - r\vartheta'_r\vartheta''_{\theta r}\vartheta'''_{\theta\theta\theta} - r^2\vartheta''_{rr}\vartheta'''_{\theta\theta r}\vartheta''_{\theta\theta} + r^2\vartheta''_{rr}\vartheta''_{\theta r}\vartheta'''_{\theta\theta\theta} \\ & - r^2\vartheta'''_{rrr}(\vartheta''_{\theta\theta})^2 + 3r^2\vartheta''_{\theta\theta}\vartheta''_{\theta r}\vartheta'''_{rr\theta} - 2r^2\vartheta'''_{r\theta\theta}(\vartheta''_{r\theta})^2 = 0. \end{aligned} \quad (2)$$

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This equation was derived after using the stationary phase method to approximate the point spread function of an imaging system with a circular aperture. They also showed that $f(r, \theta)$ must satisfy an axial symmetry condition given by

$$f(r, \theta) = -f(r, \theta + \pi). \quad (3)$$

It was assumed that the condition for extended depth-of-field has a separable solution $\vartheta(r, \theta)$ that could be written as

$$\vartheta(r, \theta) = R(r)\Theta(\theta) = -\alpha' r^m \theta^n, \quad (4)$$

where α' is a constant. It was stated that when this term $\vartheta(r, \theta)$ is substituted into equation (2), the resultant index equation which m and n must satisfy is

$$m^2 n^2 - 2m^2 n - 3mn^2 + 6mn - 4m - 8n - 8 = 0, \quad (5)$$

but, as Muyo and Harvey pointed out, the correct index equation which m and n should satisfy is given by

$$2n^2 + mn - 4n - m + 2 = 0. \quad (6)$$

For $f(r, \theta)$ to satisfy the axial symmetry, equation (3), n must be an odd positive integer, which implies that m must have a negative value. A negative value for m is problematic, as it would always result in a phase singularity at $r=0$. This phase singularity would make any solution obtained by this approach physically unrealizable. Thus another approach to solve equation (2) is necessary.

2. Optimum circular phase plate to extend the depth of field

As was previously done in [1], we ignore the second term in equation (1) due to its negligible effect on the extension of the depth of field. Thus we could write

$$f(r, \theta) \cong \vartheta(r, \theta). \quad (7)$$

To obtain $\vartheta(r, \theta)$ that satisfies the condition for extended depth of field, equation (2), we expand it as a weighted sum of Zernike polynomials:

$$\vartheta(r, \theta) = \alpha \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{nm} U_{nm}(r, \theta), \quad (8)$$

where α is a constant which determines the thickness of the phase plate. To ensure that $\vartheta(r, \theta)$ satisfies the axial symmetry condition, equation (3), we restrict the above summations to odd orders of $n-2m$. We also limit these summations to fifth degree Zernike polynomials U_{nm} as is usually done in aberration analysis. We substitute equation (8) into equation (2) and use the 'simulated annealing' optimization method [2, 3] to obtain the expansion coefficients a_{nm} that minimize the magnitude of the resultant equation. The expansion coefficients of the optimum extended depth of field (EDF) phase plate, shown in figure 1, are given in table 1. The thickness of the EDF phase plate, α , affects the tradeoff between depth-of-field extension and irreversible blurring due to possible nulls in the modulation transfer function of

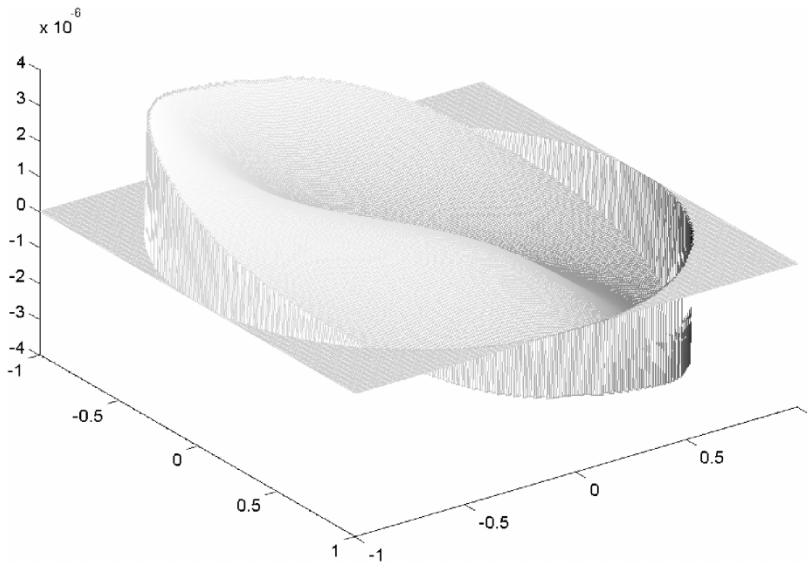


Figure 1. Profile of EDF phase plate.

Table 1. Expansion coefficients of the optimum circular EDF phase plate.

| Coefficient | a_{10} | a_{11} | a_{30} | a_{31} | a_{32} | a_{33} | a_{50} | a_{51} | a_{52} | a_{53} | a_{54} | a_{55} |
|-------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Opt. value | -0.67 | -0.15 | 0.07 | 0.13 | -0.42 | 0.10 | 0.03 | -0.04 | -0.05 | 0.08 | -0.05 | 0.0 |

the imaging system. We found the optimum value for the same imaging system used in [1] to be $\alpha = 3.8E - 6$ metres.

3. EDF phase plate performance

A quantitative way to show the extension of depth of field in a given system is to plot the angles in Hilbert space between its in-focus and out-of-focus point spread functions for different parameter values. In figure 2 we show these angles for the same imaging system used in [1]. From figure 2 we note that all the angles shown, which correspond to different defocus parameter values, become smaller when our EDF phase plate is introduced at the pupil of an imaging with a clear circular aperture. Thus the PSF of an imaging system with our EDF phase plate at its exit pupil varies less with defocus compared to the PSF of a similar standard system.

To demonstrate the extension of the depth of field, we compare two sets of computer-simulated images of a spoke target for different defocus parameter values. The first set (left column of figure 3) is obtained using a standard incoherent imaging system similar to the one used in [1] and the second set (right column of figure 3) is obtained using the same system, but with our EDF phase plate at its exit pupil. From figure 3 we note that the variation in the image quality with defocus is much less with the EDF phase plate system indicating a much increased depth of field.

The quantitative improvement in depth of field in a specific imaging system is determined by the value of α used in equation (8). On comparing figure 2 and figure 3 with figure 5 in [1] and figure 9 in [1], respectively, we note that the EDF performance of our new phase plate is considerably better than the sub-optimum phase plate derived in [1].

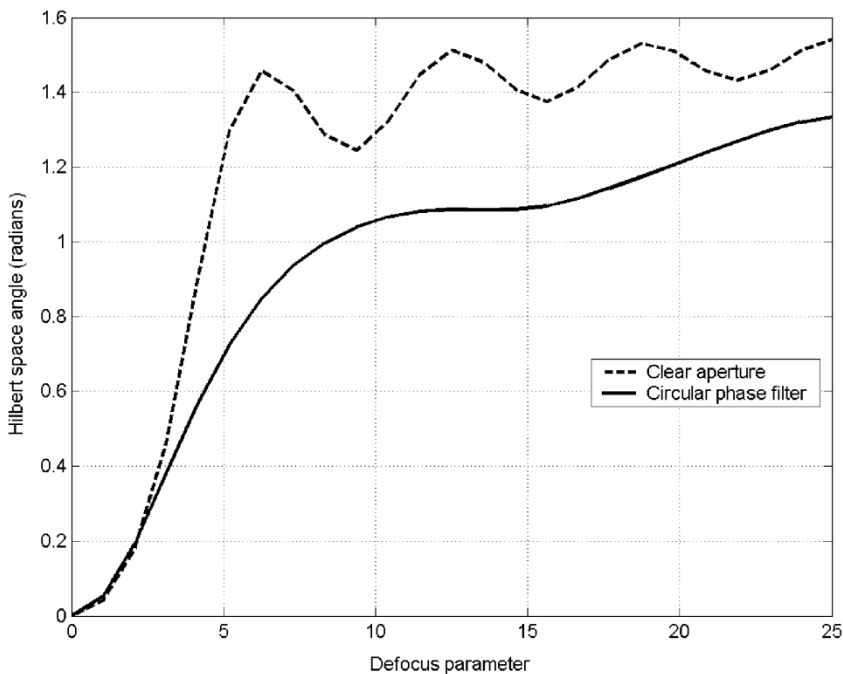


Figure 2. Hilbert space angles using a clear aperture, and using the EDF phase plate.

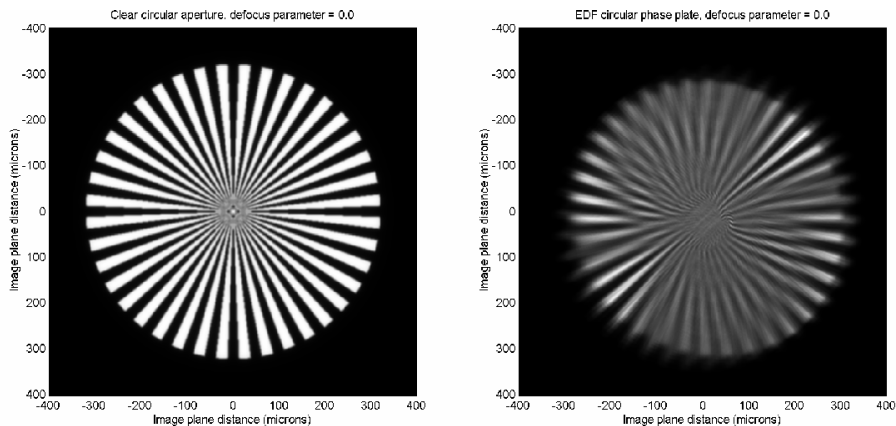


Figure 3. Defocused images using a clear aperture, and using the EDF phase plate.

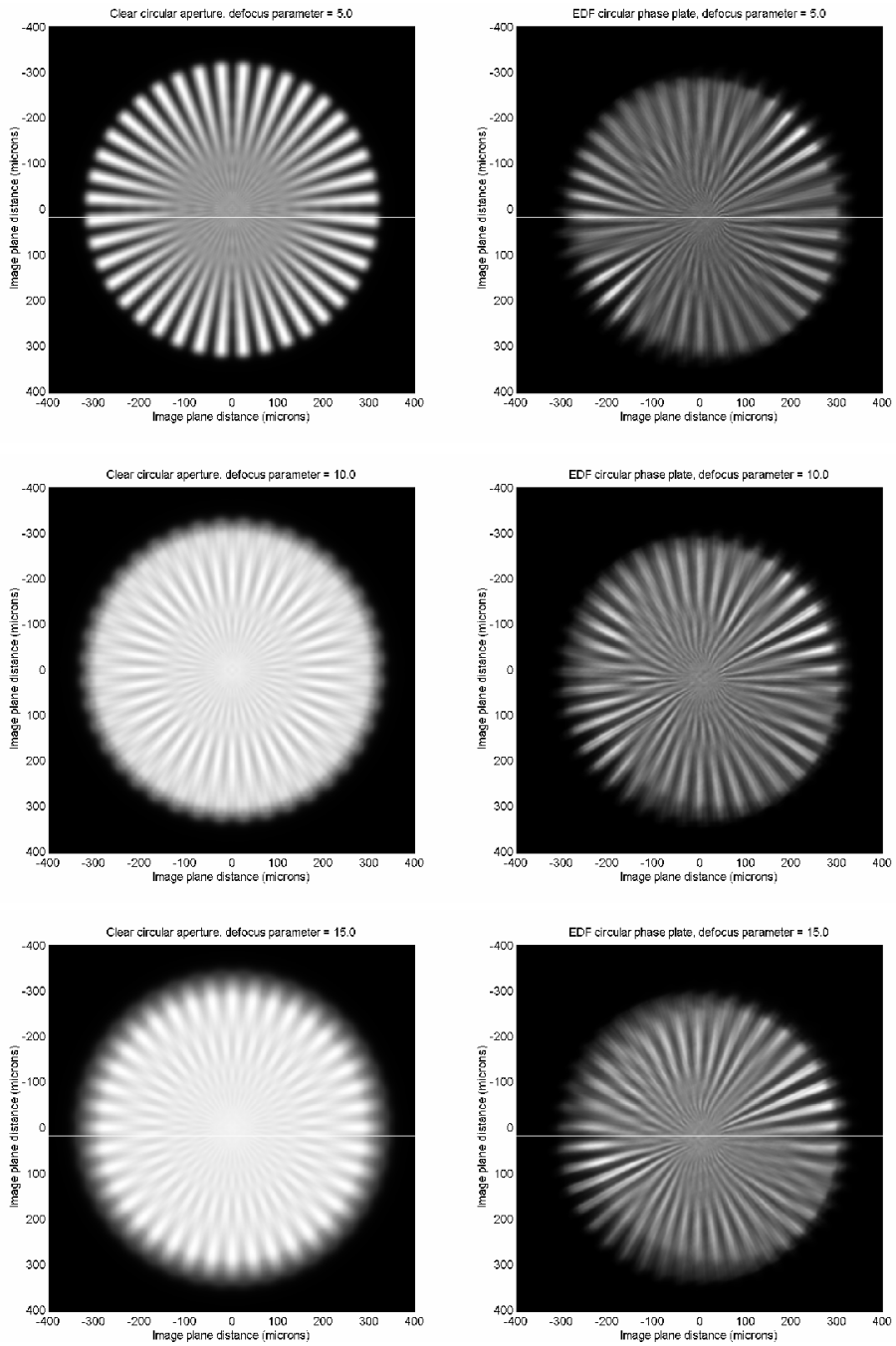


Figure 3. Continued.

4. Conclusions

We identified an error in the derivation of the EDF phase plate reported in [1]. This error makes the earlier reported extension of depth-of-field sub-optimum rather than invalid. In this comment, we obtained an optimum EDF phase plate and repeated the relevant results.

Acknowledgment

We acknowledge the usage of the Adaptive Simulated Annealing (ASA) optimization code developed by Lester Ingber.

References

- [1] S.S. Sherif, E.R. Dowski, W.T. Cathey, *J. Mod. Opt.* **51** 1191 (2004).
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