Table 1 ERROR BEHAVIOUR

<table>
<thead>
<tr>
<th>Impulse response identified</th>
<th>True value $d_1 = 3, d_2 = 1, d_3 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Errors in determination of output signal $g$</td>
<td>$e_1 = 0.1, e_2 = -0.1, e_3 = 0.1$</td>
</tr>
</tbody>
</table>

- 'good' input signal
  - $s = [10, 1]$ cond. $(s) \geq 0.995$
- 'bad' input signal
  - $s = [1, 10]$ cond. $(s) \geq 0.995$
- 'good' regularisor
  - $h_1 = 3:01, h_2 = 0:989$
  - $h_3 = 0:1$, $h_4 = -0:1$
- 'bad' regularisor
  - $h_1 = 3:01, h_2 = 0:989$
  - $h_3 = 0:1$, $h_4 = -0:1$

Inaccurate determination of $g$ affects the estimation of true values of $h$ in two ways:

(a) It causes the spurious sample $h_i$ to appear in the estimated impulse response. The magnitude of this sample depends on both the cond $(s)$ and the parameter of regularisation $a$.

(b) It causes the inaccuracies in the determination of the true samples $h_i$ and $h_j$. The degree of inaccuracy depends on cond $(s)$ only.

The same conclusions are true for $s$ and $g$ being longer sequences. In this case the number of spurious samples that appears in the estimated impulse response is $k - 1$.

To illustrate the behaviour of the error in the determination of the impulse response $h$, four combinations of a 'good' or 'bad' signal $s$ and the regularisator are taken into account and the results of computations are presented in Table 1. These results are self-explanatory and require no further comment.

A. DYKA
22nd May 1989
Institute of Telecommunications
Technical University of Gdańsk
80-852 Gdańsk, Poland

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FREQUENCY-DEPENDENT ANALYSIS OF FINLINE MIXED-TYPE OFFSET JUNCTIONS

Indexing terms: Microwave devices and components, Frequency-domain analysis

The frequency-dependent characteristics of finline mixed-type offset junctions are analysed by employing a mode-matching technique in conjunction with a spectral-domain method. Some numerical results are presented.

Introduction: Finlines have been widely applied in millimetre-wave integrated circuits. The general finline step junction as shown in Fig. 1 is the most frequently encountered structure in the circuit design. Therefore, it is important to develop analytical techniques to characterise the step junction. However, up to this date most of the theoretical work has been concerned with the analysis of coaxial ($\alpha = 0$) or offset ($\alpha \neq 0$) step junctions, only a relatively small effort has been made to analyse mixed-type offset ($\alpha > 0$, $\alpha < 0$) step junctions, and their frequency-dependent behaviour has not been reported.

This letter presents a rigorous and efficient hybrid-mode approach for the analysis and characterisation of finline mixed-type offset junctions. The approach is based on the mode-matching technique and the spectral-domain method. In the result the formulation of the generalised scattering matrix which represents the propagating as well as the evanescent hybrid-mode scattering properties of this kind of finline discontinuity is arrived at. The frequency-dependent behaviour of a unilateral finline mixed-type offset junction is illustrated.

Theoretical approach: As a matter of fact, the major complexity of the analysis of a finline mixed-type offset junction lies in a mixed-type boundary which consists of two conducting obstacle surfaces $S_1$ and $S_2$ for lines I and II, respectively, besides a common aperture surface $S_0$. For the treatment of this kind of boundary problem, an auxiliary finline section (line III) is introduced, as shown in Fig. 2. Next, the EM fields in each homogeneous line are expanded in terms of hybrid modes $HE_x$ and $EH_y$, as follows:

$$E_{10}^0 = \sum_{x} V \times V \times HE_{x}^0 - j\omega V \times HE_{y}^0$$
$$H_{10}^0 = \sum_{x} j\omega V \times HE_{x}^0 + V \times HE_{y}^0$$

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Fig. 1 General finline step junction
- a. Longitudinal-section
- b. Cross-section of unilateral finline
- c. Cross-section of bilateral finline

Fig. 2 Equivalent model for finline mixed-type offset junction
where the magnetic and electric vector potentials are expressed by
\[ \Pi_{m}^{\text{in}} = a_{m}^{*} \phi_{m}^{\text{in}}(x, y), b_{m}^{*} \exp(\pm iK_{m}z) \]
\[ \Pi_{m}^{\text{out}} = a_{m}^{*} \phi_{m}^{\text{out}}(x, y), b_{m}^{*} \exp(\pm iK_{m}z) \]  \hspace{1cm} (2)
where the normalisation coefficient \( a_{m} \) is calculated so that the unknown eigenmodes \( a_{m} \) and \( b_{m} \) represent the incident and reflected mode variables, respectively. The eigenvalue \( \lambda_{m} \) and the eigenfunctions \( F_{m}(\xi, \eta) \) and \( P_{m}(\xi, \eta) \) can be determined by the spectral-domain method.\(^6\)

The boundary conditions on the \( z = 0 \) and \( z = 0^+ \) discontinuous planes are
\[ \Phi_{m}^{i} = \Phi_{m}^{o} \quad \text{on} \quad S_{z} \]  \hspace{1cm} (3a)
\[ H_{m}^{i} = H_{m}^{o} \quad \text{on} \quad S_{z} \]  \hspace{1cm} (3b)
\[ E_{m}^{i} = 0 \quad \text{on} \quad S_{z} \]  \hspace{1cm} (4a)
\[ E_{m}^{o} = 0 \quad \text{on} \quad S_{z} \]  \hspace{1cm} (4b)
where subscript \( i \) indicates the transverse fields. By taking the cross-products of both sides of eqns. 3a-4b with proper vector weights and then integrating them over the proper areas, the matching equations are transformed into a set of linear equations. As the auxiliary finline section is reduced to zero, the generalised scattering matrix of the mixed-type offset junction can be directly constructed from this system of linear equations as
\[ \begin{bmatrix} \Phi_{m}^{i} \\ H_{m}^{i} \\ E_{m}^{i} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \Phi_{m}^{o} \\ H_{m}^{o} \\ E_{m}^{o} \end{bmatrix} \]  \hspace{1cm} (5)
with
\[ [S_{11}] = ([Q][R] + [D^{1}]^{-1})([Q][R] - [D^{1}]) \]
\[ [S_{12}] = ([Q][R] + [D^{1}]^{-1})([Q] - [Q][R][U] - [S_{21}]) \]
\[ [S_{21}] = ([Q] - [Q][R][U] - [S_{11}]) \]
\[ [Q] = ([C^{10}][C^{01}]^{-1})[D^{1}] \]
\[ [R] = ([F^{10}][F^{10}]^{-1})[F^{10}] \]
where \( [U] \) is the unit matrix, and the elements for the other matrices are
\[ [C^{01}][C^{10}]^{-1} = a_{m}^{*}(\phi_{m}^{01}, \phi_{m}^{10}) \]
\[ [D^{1}] = a_{m}^{*}(\phi_{m}^{01}, \phi_{m}^{10}) \]
\[ [F^{10}] = a_{m}^{*}(\phi_{m}^{01}, \phi_{m}^{10}) \]
\[ m = 1, 2, \ldots; m = 1, 2, \ldots; s = I, II \]  \hspace{1cm} (6)
where the inner product \( (e, h) \) is defined as \( \int e \times h \, dx \). The eigenfields \( \Phi \) and \( H \) can be easily derived from eqs. 1. In consideration of the availability of the Fourier transforms of the eigenfields, the inner products in the space domain are transformed into the spectral domain using the Parseval theorem so that only one-dimensional integrations with respect to \( y \) need to be solved.

Numerical results: To confirm the validity of our approach, a unilateral finline mixed-type offset junction with \( w_{1}/b = 0.3, d_{1}/b = 0.15, d_{2}/b = 0.4, \alpha/b = 0.25, h/b = 0.0357, a = 2b \) and \( s_{1} = 2 \times 2.22 \) has been analysed and its frequency-dependent behaviour is depicted in Fig. 3. As can also be seen from Fig. 3, the dominant mode scattering matrix is unitary, as expected according to the theory, since only the dominant modes on both sides of the junction are propagating. Further computations show that the numerical results for different offsets converge smoothly to the expected results for the zero offset.

Conclusions: The hybrid-mode approach which combines the spectral-domain method with the mode-matching technique has been found to be the most powerful tool for the characterisation of finline mixed-type offset junctions. The approach is not only mathematically exact but also numerically efficient. The generalised scattering matrix representation for the single junction discontinuity makes it possible, and easy, to model accurately more-complicated finline mixed-type offset step discontinuities.

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JIA SHENG HONG
JUN MING SHI
Department of Radio Engineering
Fuzhou University
Fujian, People's Republic of China

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