Statistical Hypothesis Testing for assessing Monte Carlo Estimators: Applications in Image Synthesis

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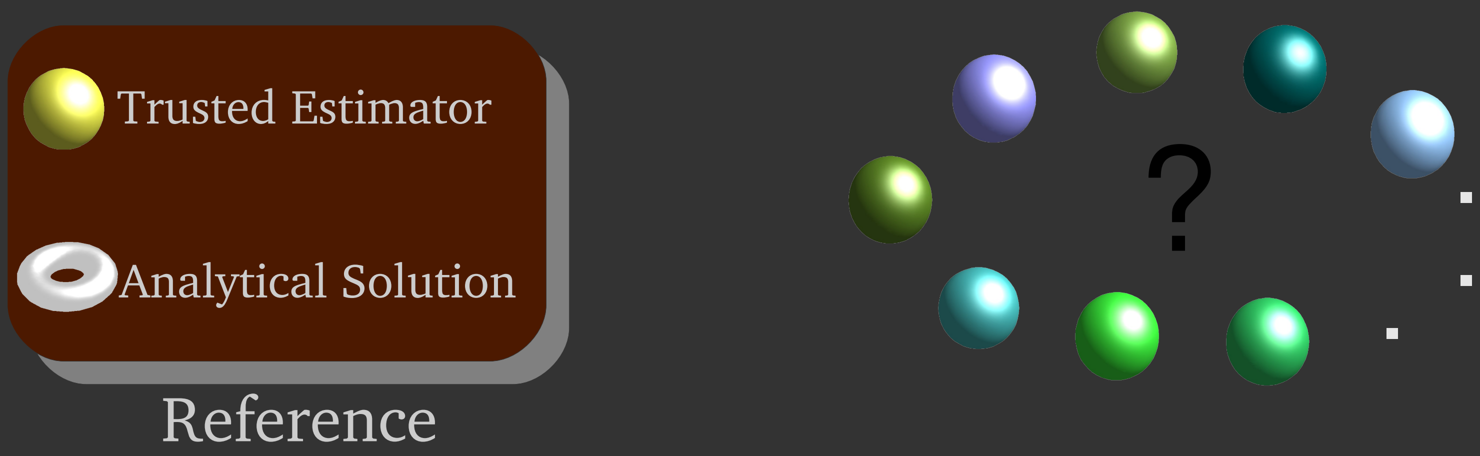
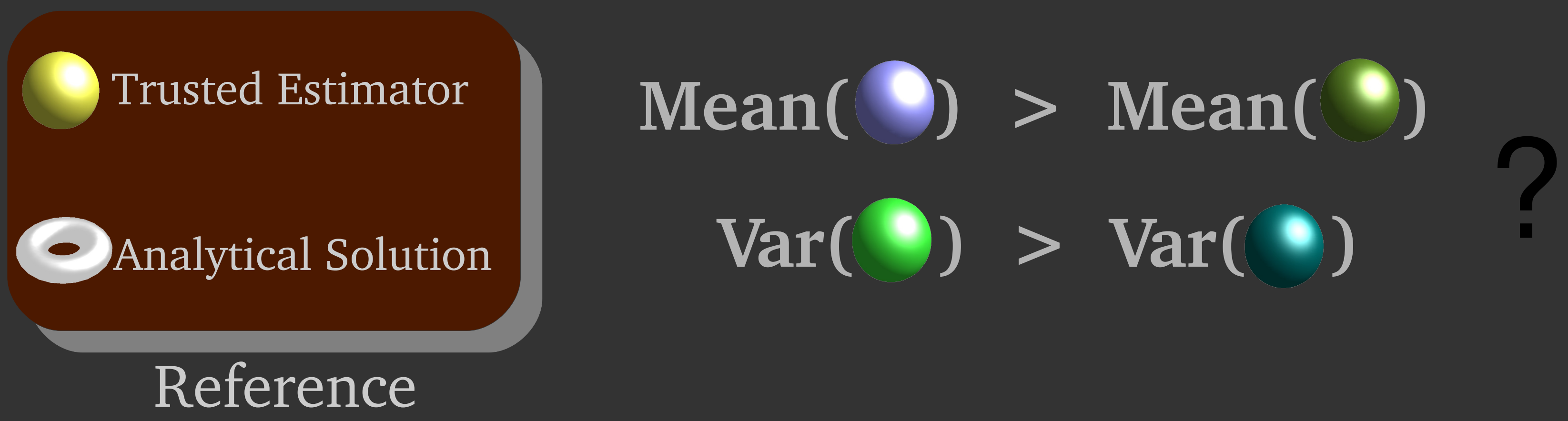
Assessing Estimators



Assessing Estimators

Typically compare 1st and/or 2nd order statistics

i.e. Mean and Variance



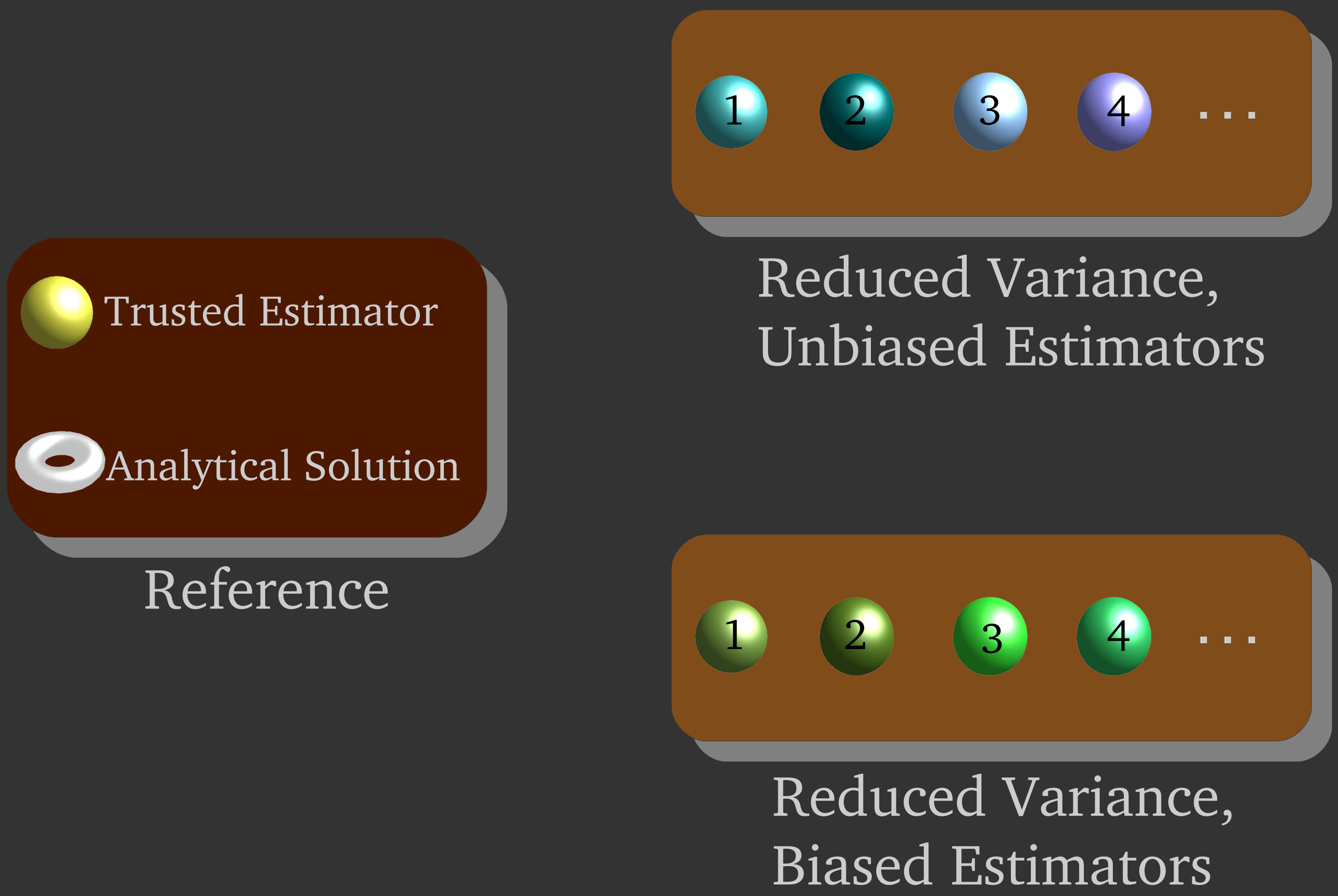
Assessing Estimators- Image Synthesis

- Cost
  - time
  - number of samples
- Mean
  - difference images
  - inspecting convergence plots
- Variance
  - inspecting image noise

Assessing Estimators- Image Synthesis

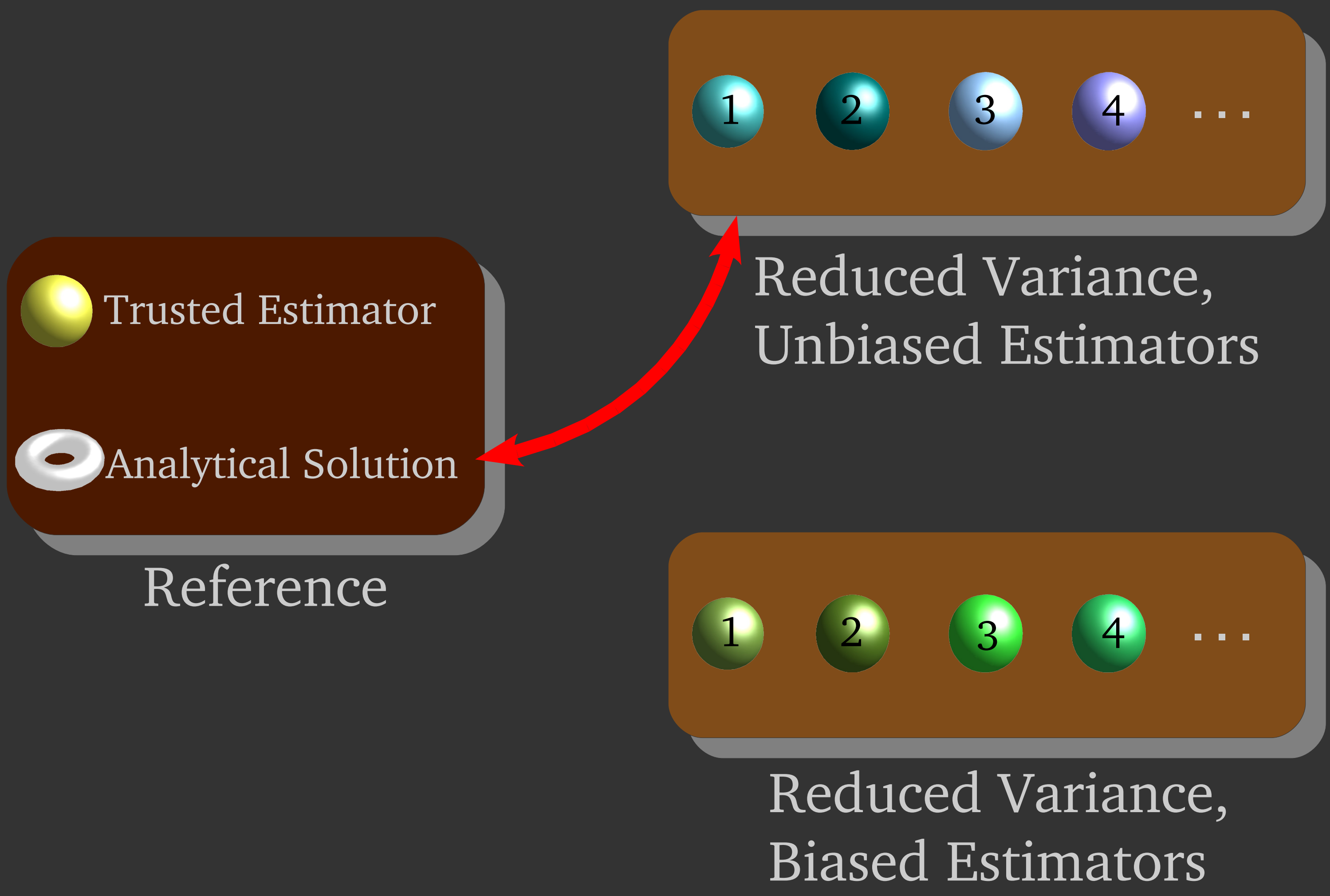
- Drawbacks (current techniques)
  - subjective
  - weakly quantitative
  - comparing variance plots- large number of estimates
  - difficult, often impossible, to automate

Typical Classes of MC Estimators in Image Synthesis



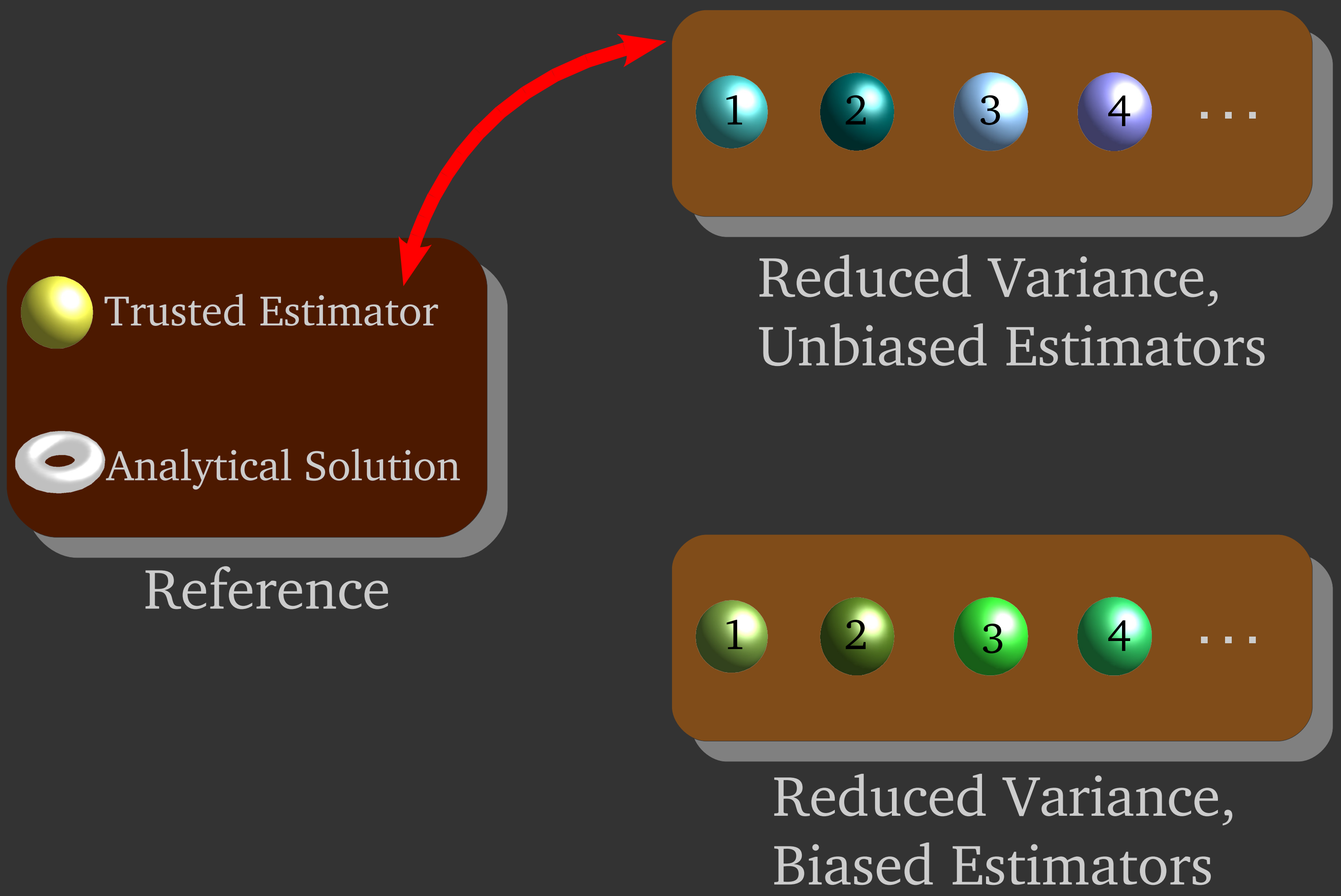
Verifying Absence of Bias

1. Estimator vs Analytical Solution



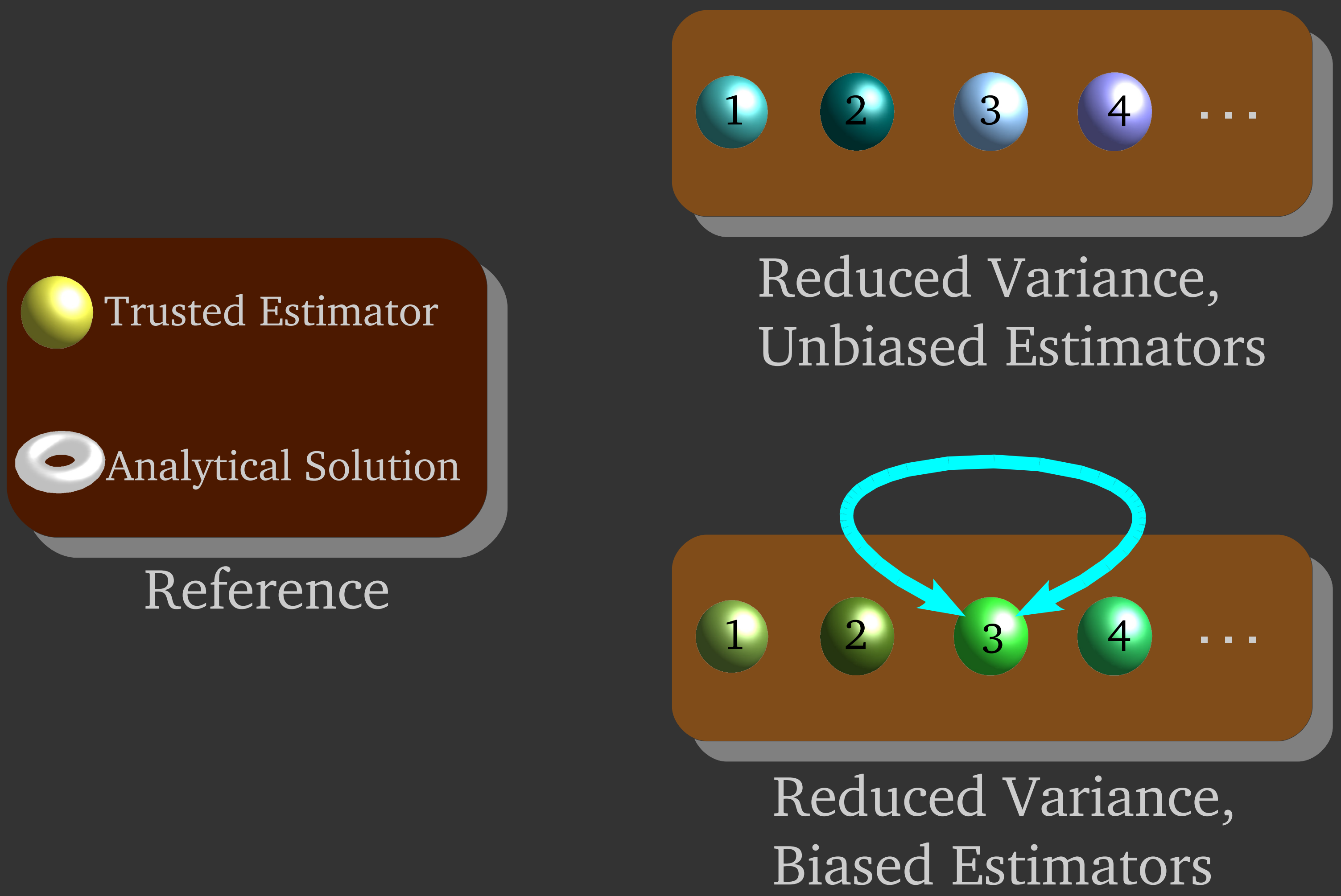
Verifying Absence of Bias

2. Estimator vs Trusted Estimator



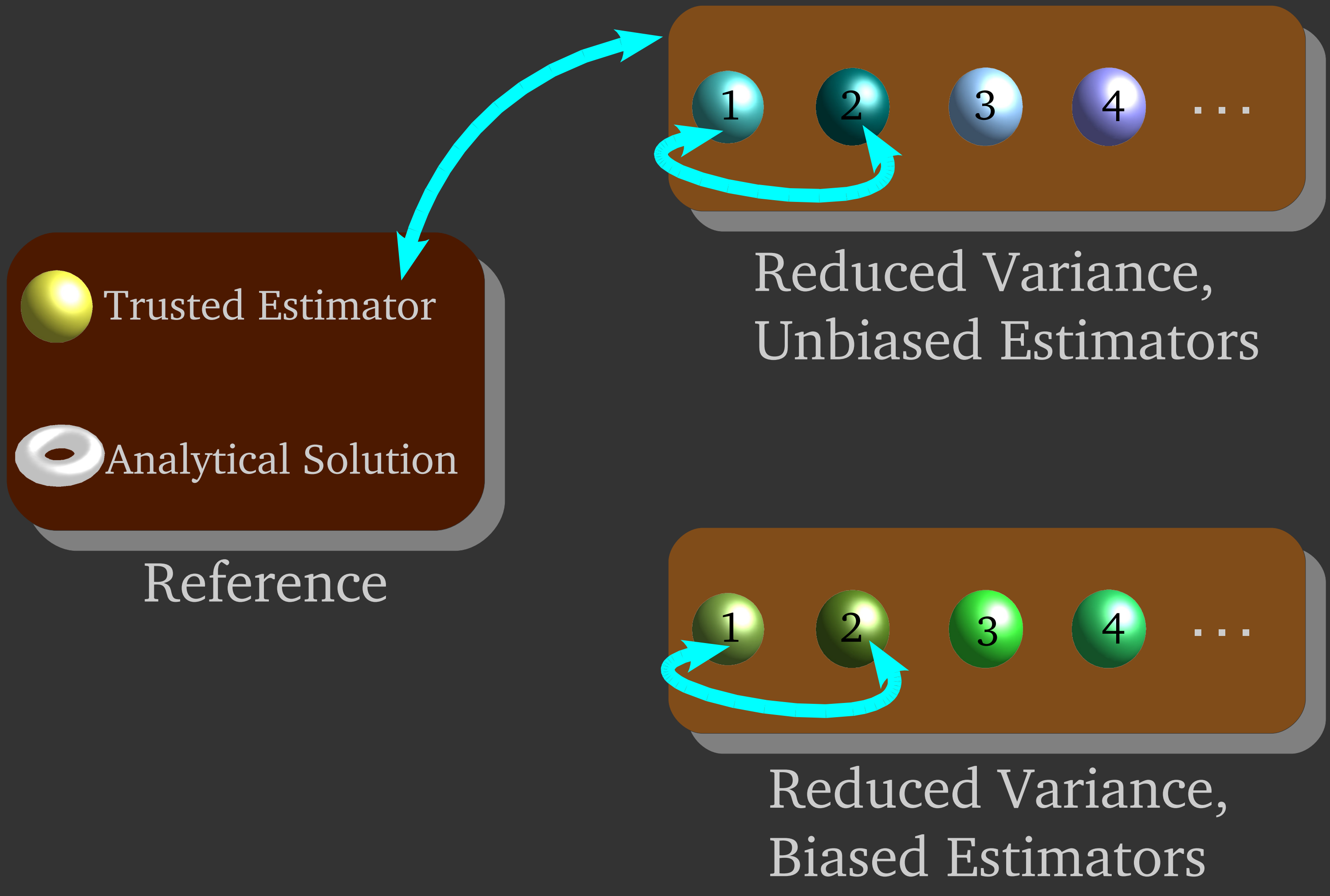
Verify Variance Acceptability

3. Verify variance acceptibility- Estimator vs Constant

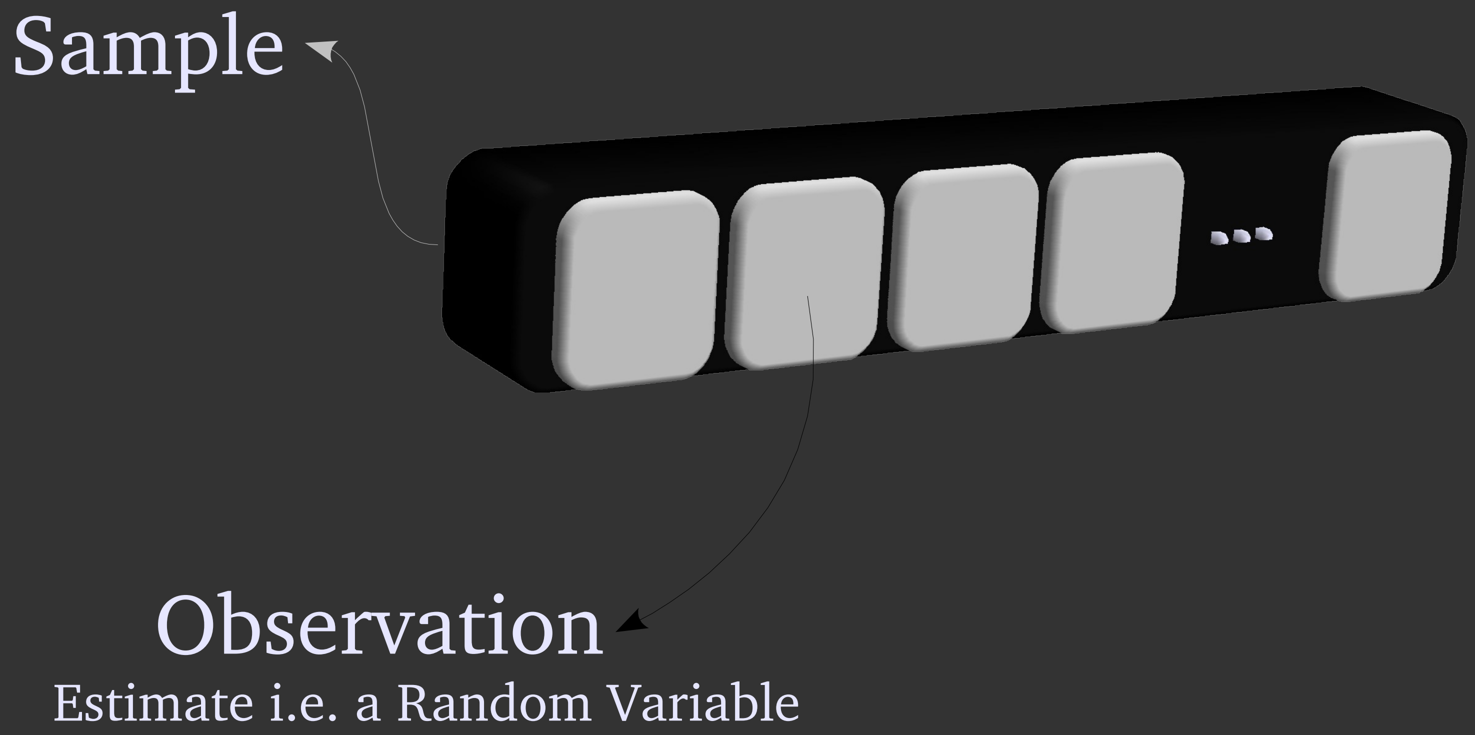


Verify Variance Reduction or Compare Variances

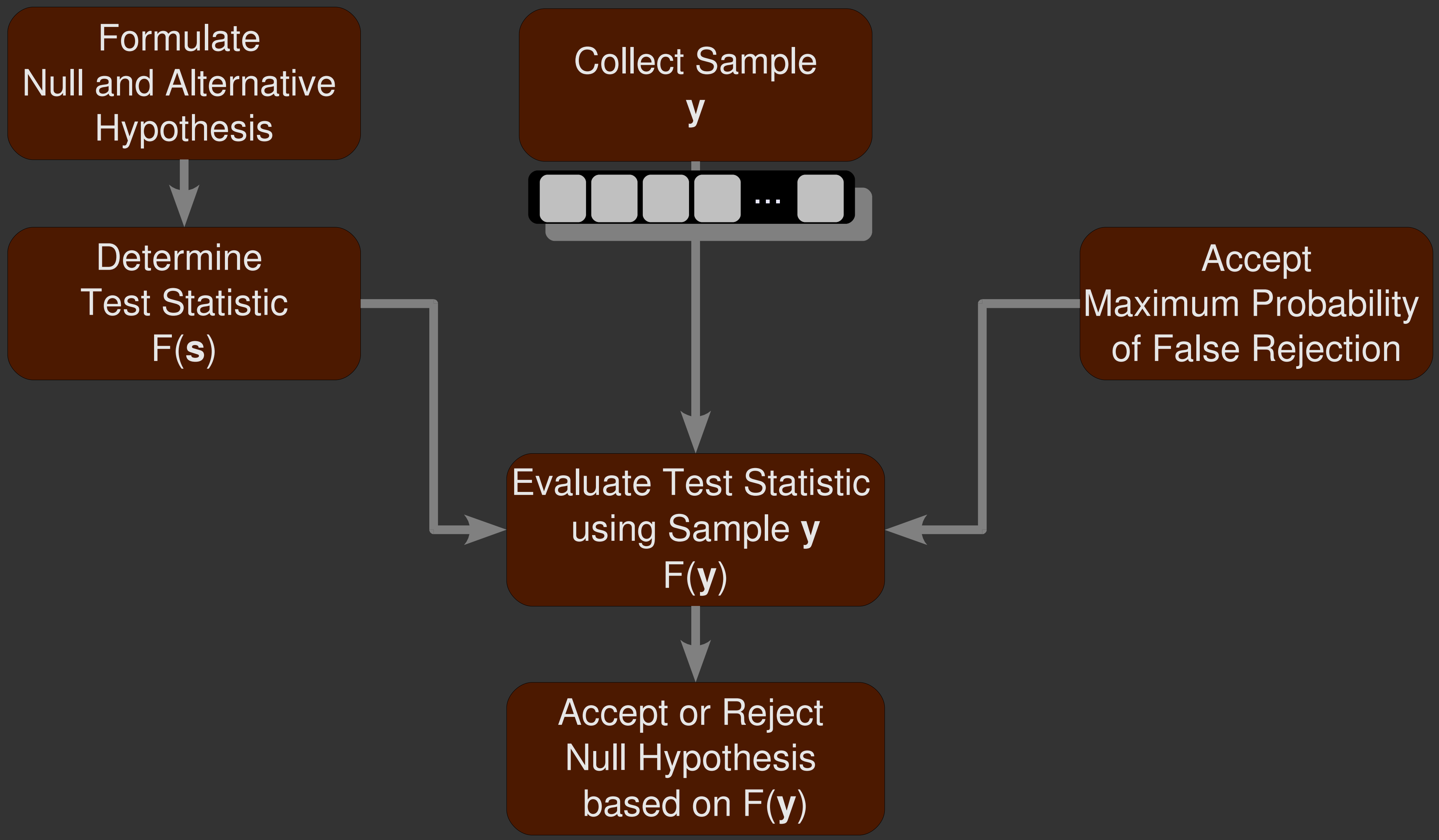
4. Estimator vs Estimator



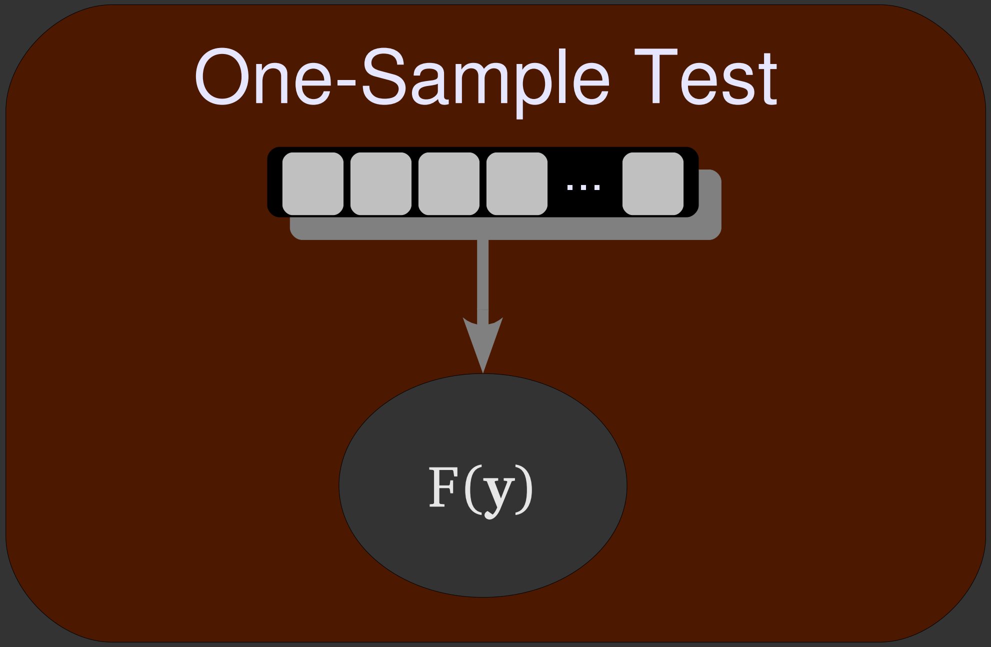
Sample: Collection of observations

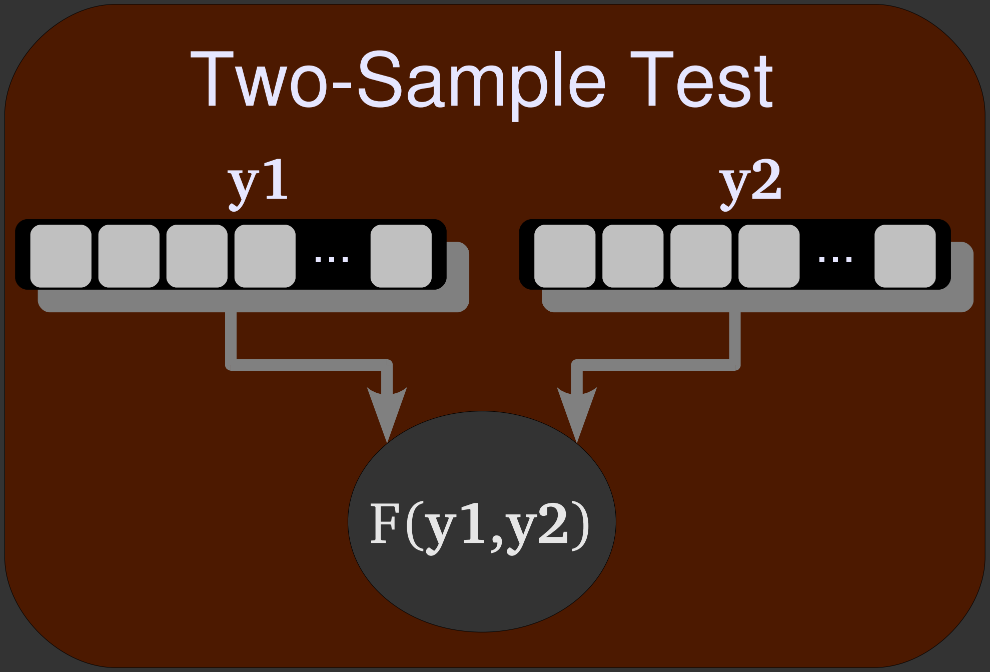


Review: Hypothesis Testing



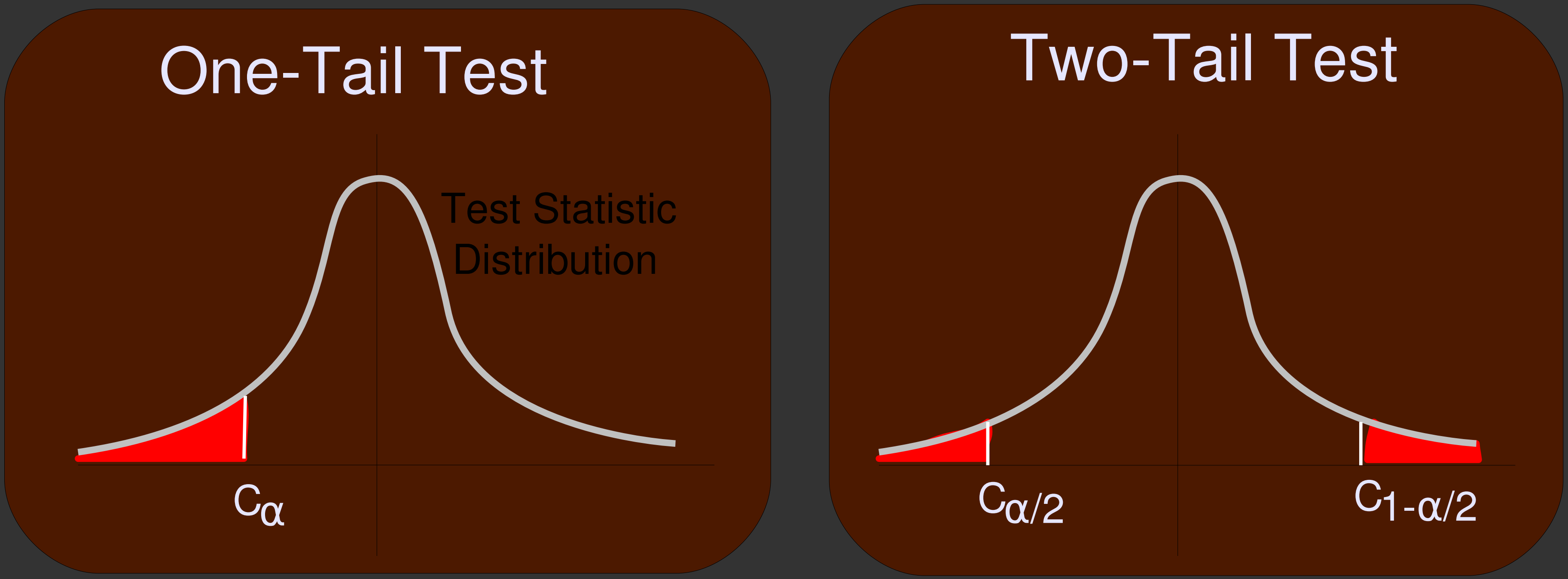
Review: One-Sample vs Two-Sample Tests





Review: Rejecting the Null-Hypothesis

- Find boundaries of rejection region
- Compute F(**y**) using the sample '**y**'
- Reject if F(**y**) falls inside rejection region



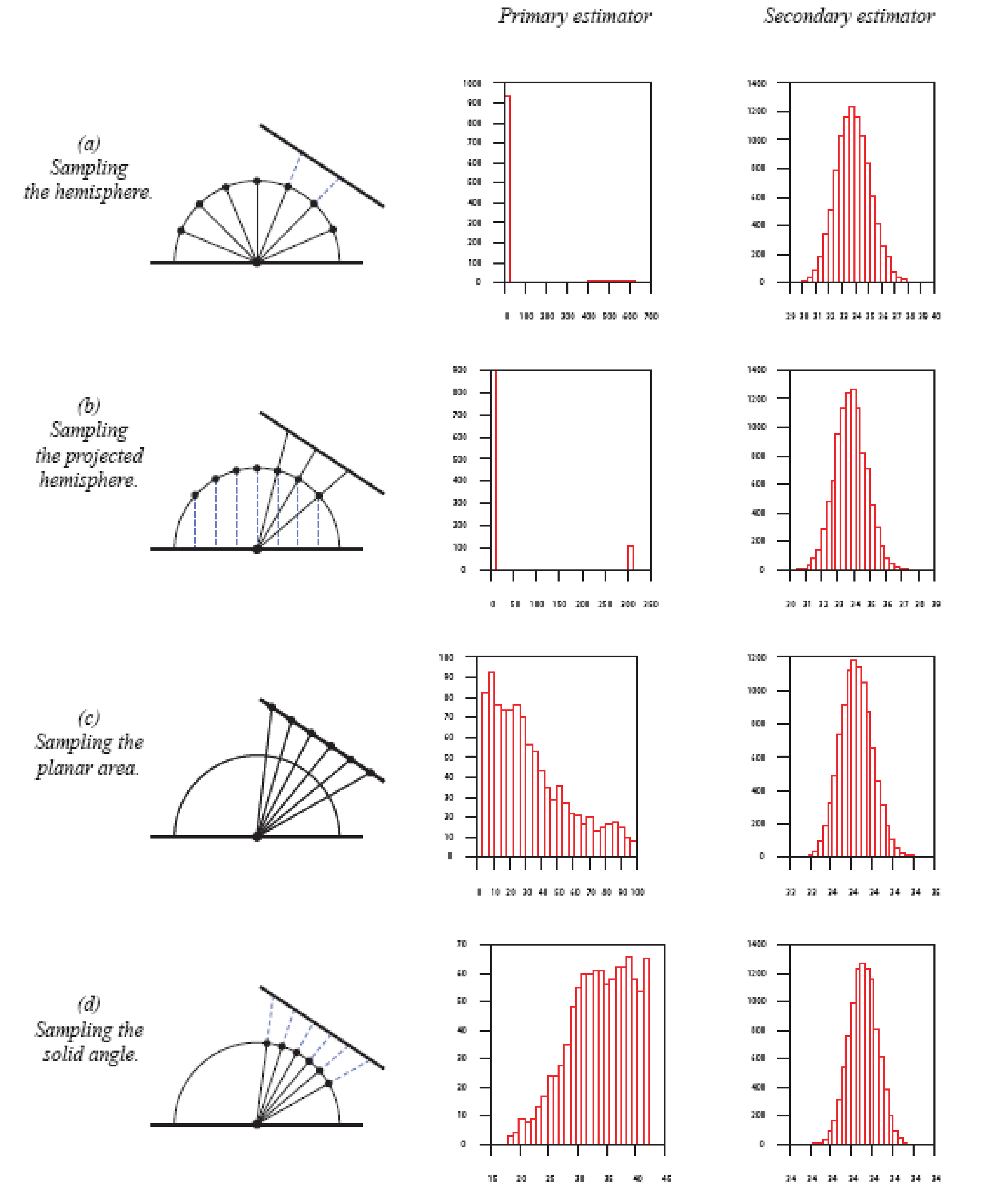
$C_\alpha = G^{-1}(\alpha)$ where G(s) is the CDF of F(s)

Tests Performed and their Test Statistics

	One-Sample Tests	**Two-Sample Tests**
Test for Mean	Test for Bias against Constant Student's t-distribution	Compare Means of Two Estimators Student's t-distribution
Test for Variance	Test that Variance is Bounded Chi-Square distribution	Compare Variances of Two Estimators F-Distribution

Setting up Hypothesis Tests

- Careful
  - Sensitive to distribution
  - most tests for normally distributed data
- Testing Estimators in Image Synthesis
  - Compare secondary instead of primary estimators



Primary estimator
Secondary estimator
(a)
Sampling
the hemisphere.
(b)
Sampling
the projected
hemisphere.
(c)
Sampling the
planar area.
(d)
Sampling the
solid angle.

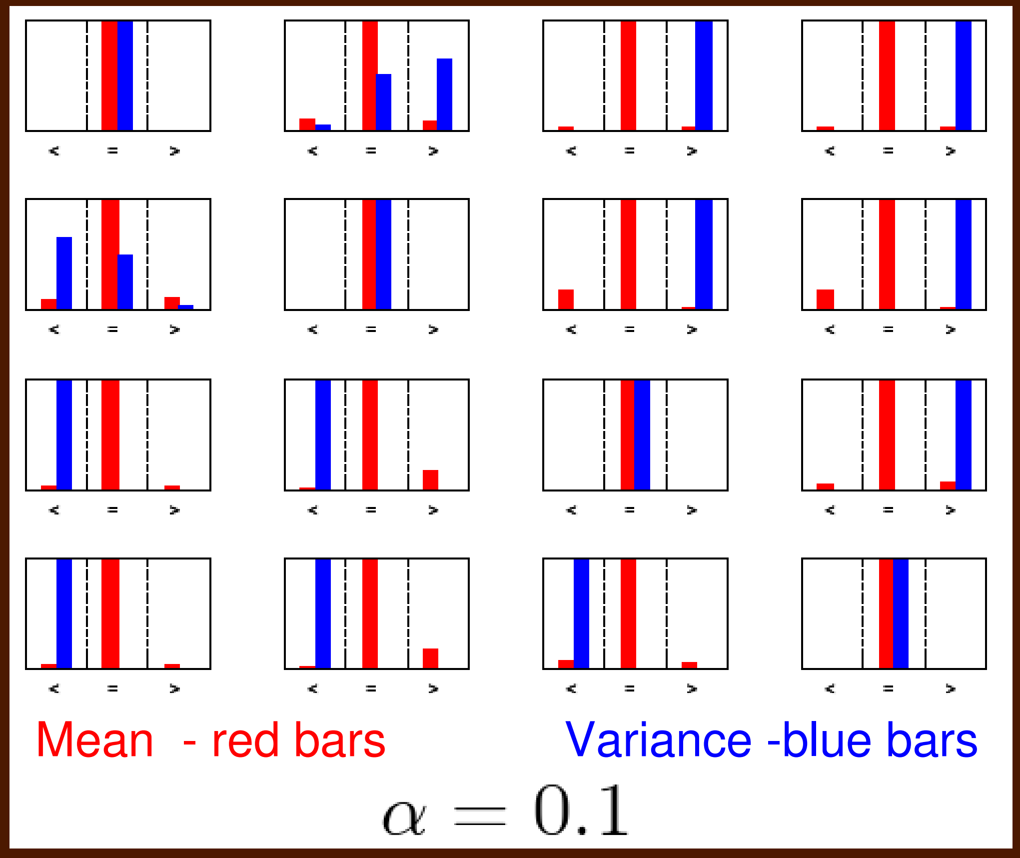
Results: Comparing Means and Variances

Uniform Hemisphere

Uniform Proj-Hemisph.

Uniform Area on Light

Uniform Solid Angle



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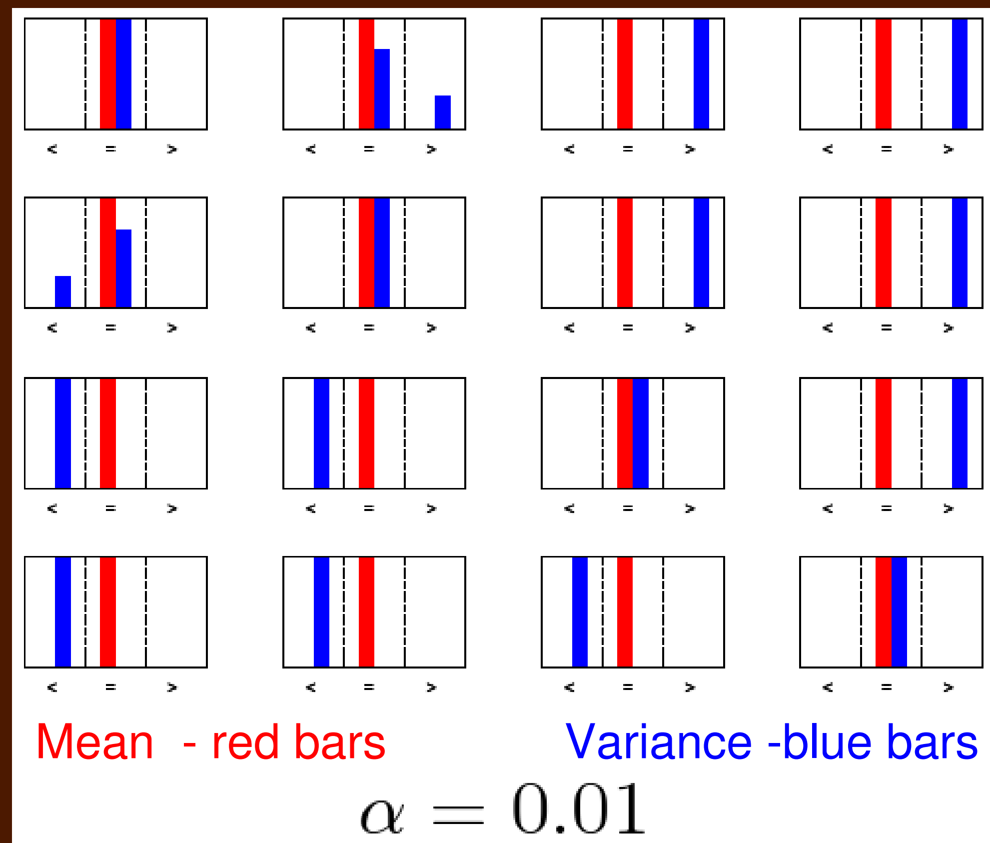
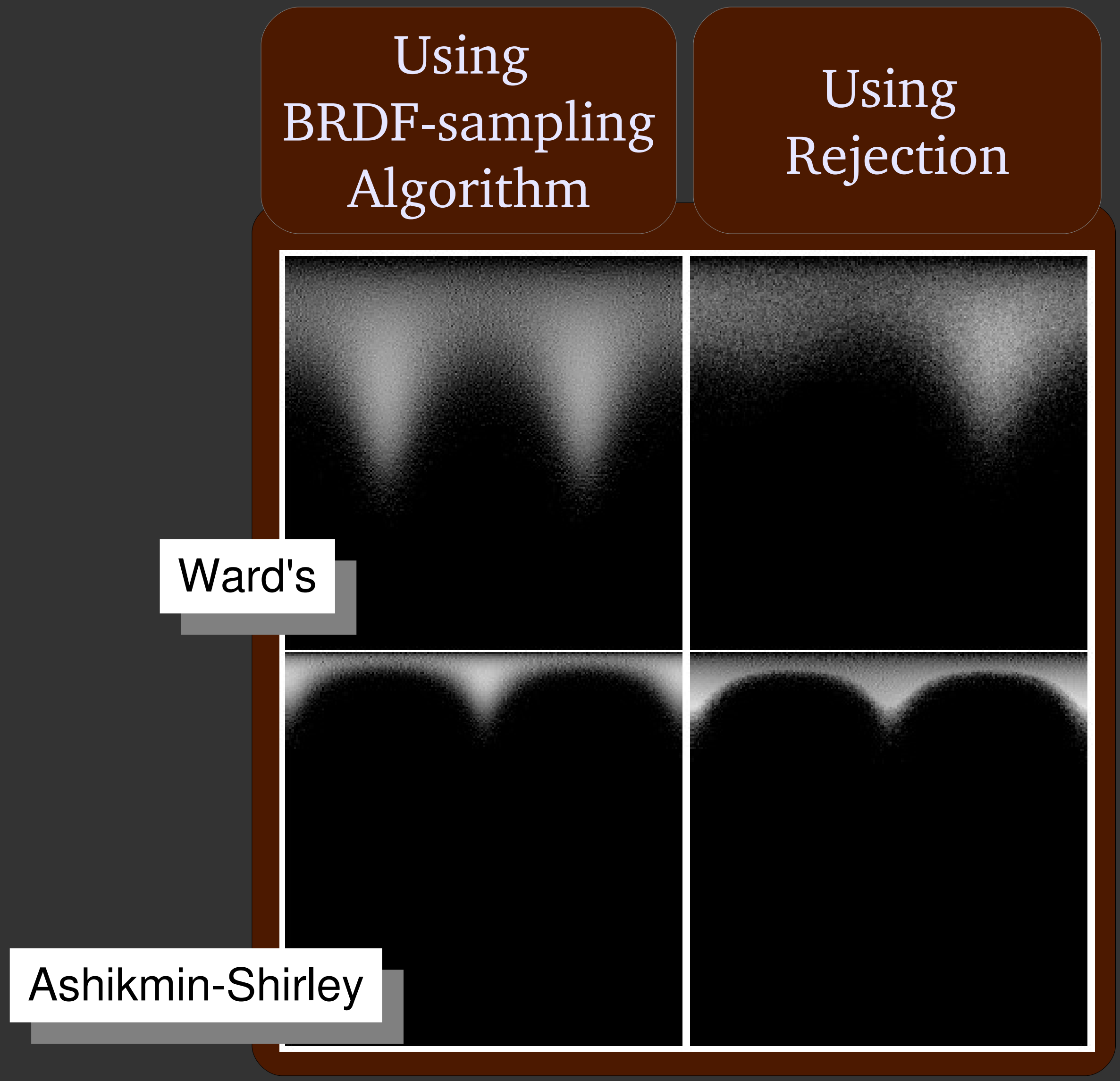


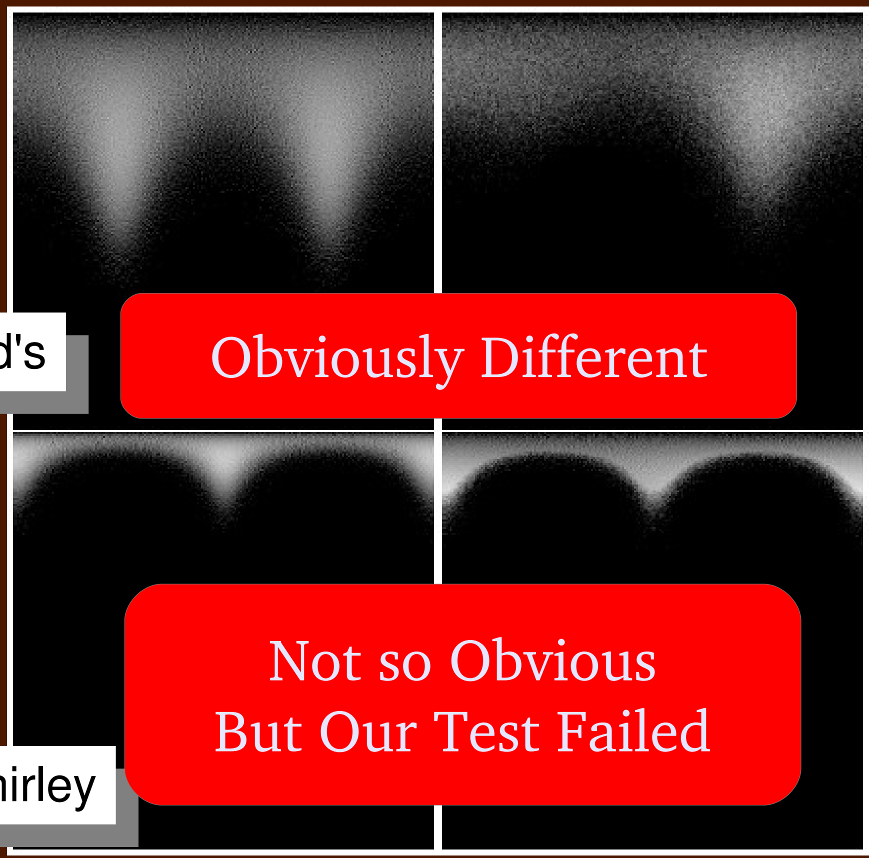
BRDF Sampling



Results – BRDF Sampling

2-Sample Goodness-of-fit (Kolmogorov-Smirnov)

	Using BRDF-sampling Algorithm	Using Rejection
Ward's		
Ashikmin-Shirley		



Obviously Different

Not so Obvious
But Our Test Failed

Irradiance due to a Triangular Illuminaire

Shading Normal

Light Source Normal

$$E(\mathbf{x}) = \int_{Area(\triangle)} L(\mathbf{x}, \mathbf{z}) \frac{\mathbf{n} \cdot \mathbf{z}}{\|\mathbf{z}\|} \frac{\mathbf{n}_{\triangle} \cdot \mathbf{z}}{\|\mathbf{z}\|^3} \, d\mathbf{y}$$

Irradiance

Light Source

Radiance

z = x - y

Irradiance due to a Triangular Illuminaire

$$E(\mathbf{x}) = \int_{Area(\triangle)} L(\mathbf{x}, \mathbf{z}) \frac{\mathbf{n} \cdot \mathbf{z}}{\|\mathbf{z}\|} \frac{\mathbf{n}_\triangle \cdot \mathbf{z}}{\|\mathbf{z}\|^3} \, d\mathbf{y}$$

- Create Erroneous Estimators
  - Omitting the cosine term for shading

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- Create Erroneous Estimators
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  - Non-uniform sampling of illuminaire

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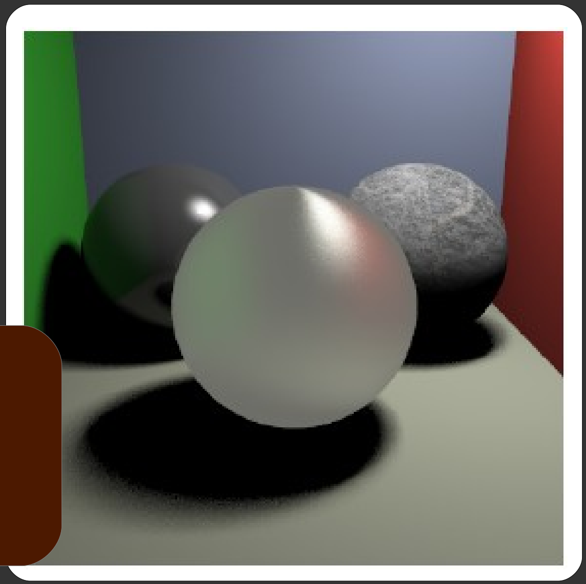
- Create Erroneous Estimators
  - Omitting the cosine term for shading
  - Non-uniform sampling of illuminaire
  - Omitting change of variables

Results – Error Detection



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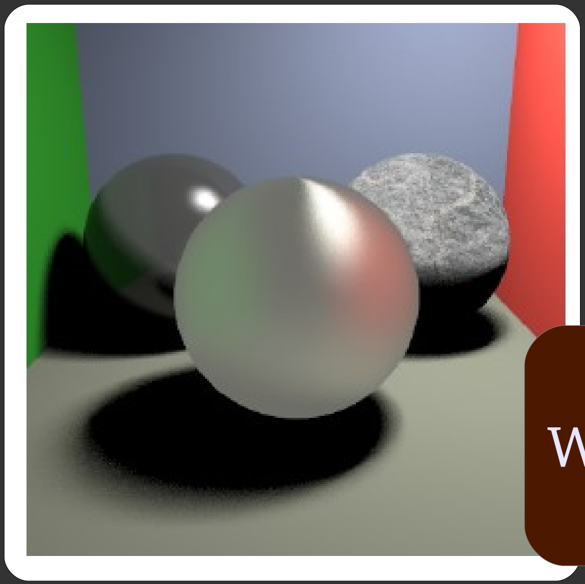
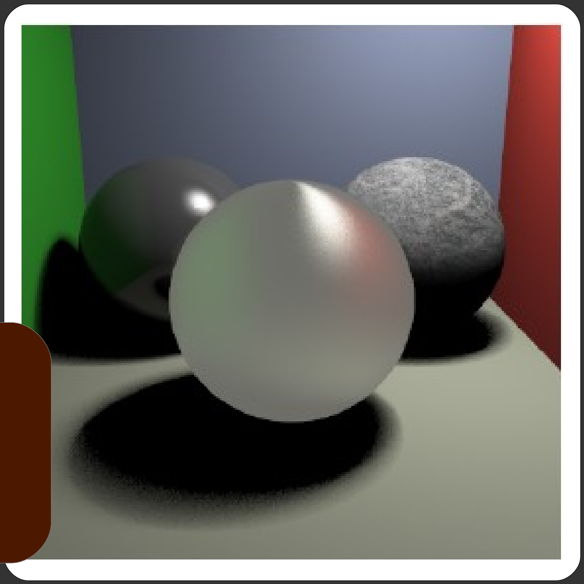
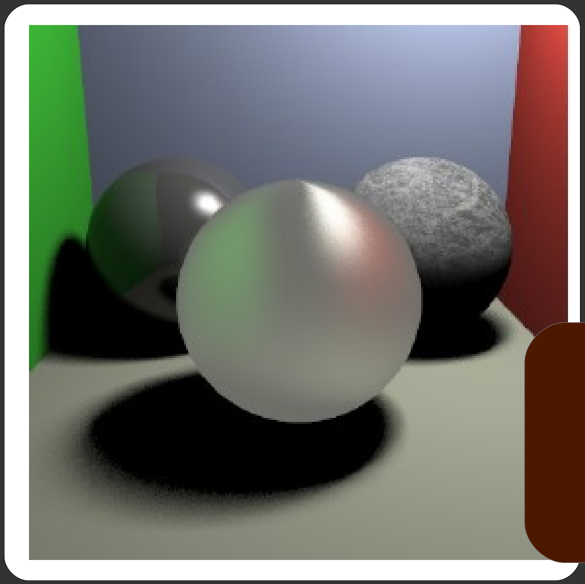


Without Cosine

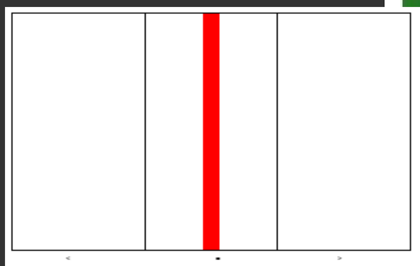
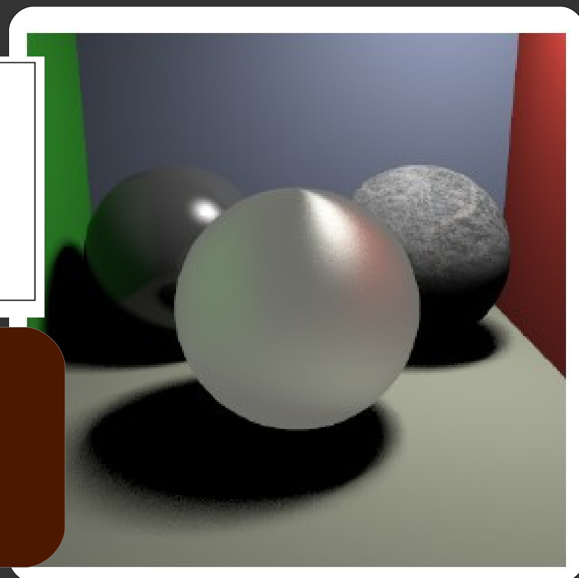


Non-uniform Sampling of Light Source



Incorrect Change of Variables

Results – Error Detection

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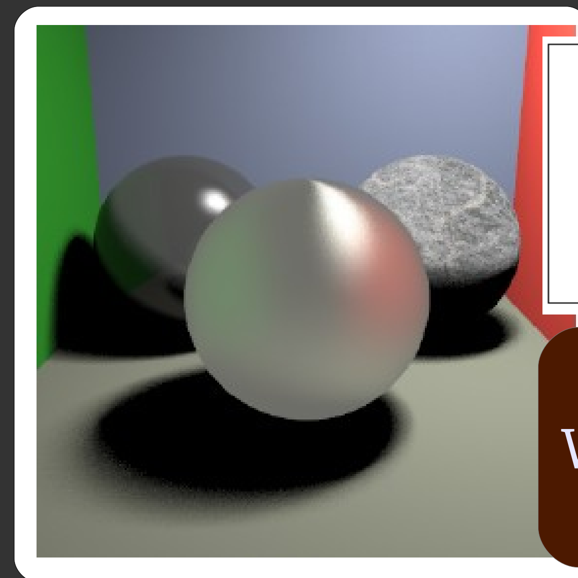
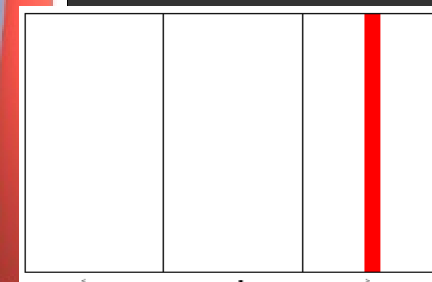
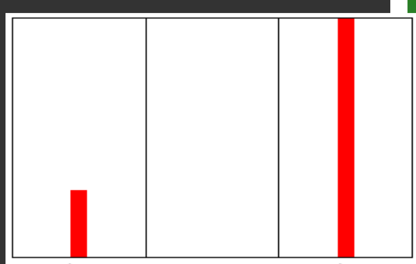
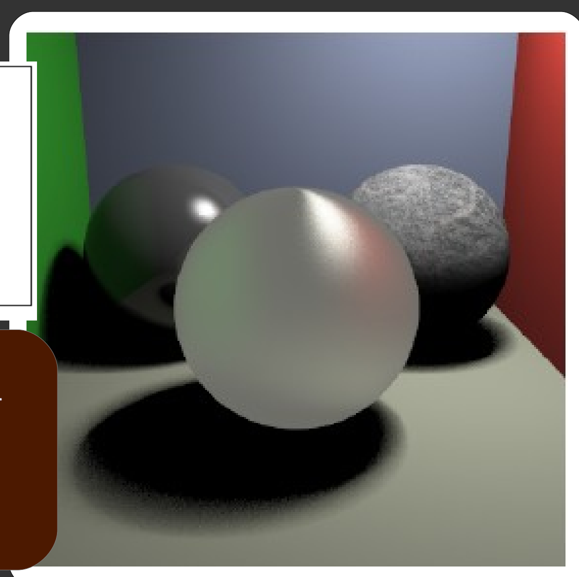



Without Cosine

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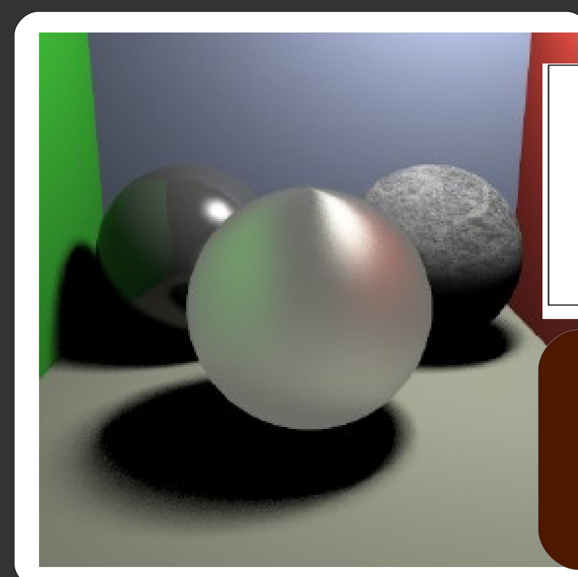
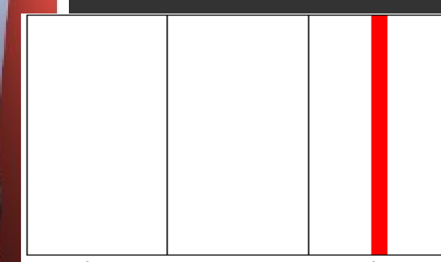

Incorrect Change of Variables

Conclusion

- Tested Estimator for Bias/acceptable Variance
- Compared Means/Variances of Estimators
- Verified BRDF Sampling
- Showed Usefulness in Detecting Errors

References

[Fisher 59]

Statistical Methods and Scientific Inference

[Neyman & Pearson 28]

On the Use and Interpretation of Certain Test Criteria for Purposes of Statistical Inference

[Freund & Walpole 87]

Mathematical Statistics