FINITE IMPULSE RESPONSE (FIR) DIGITAL FILTERS

Paolo Favaro

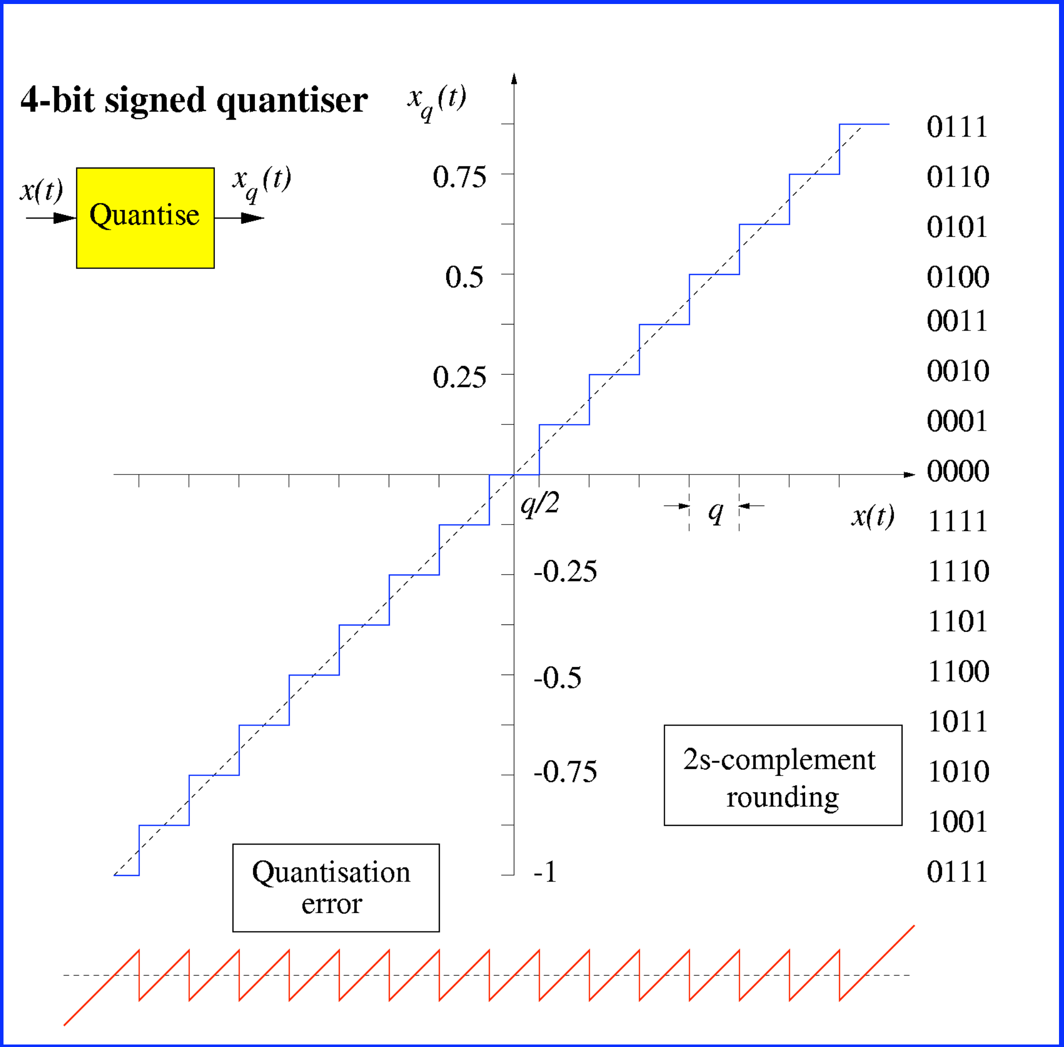
Chapter 6 of Textbook -- part III

(original material from John Thompson)

Finite Precision Effects

- Quantisation noise and effects
- Filter coefficient quantisation
- FIR Filter Examples

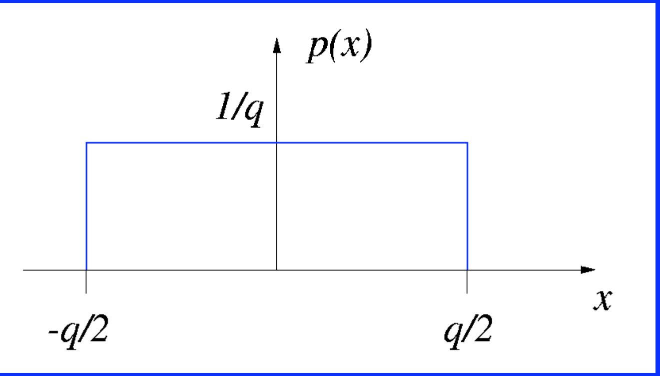
Signal Quantisation



Quantisation Noise

- Must select max quantisation level to avoid/minimise saturation
- Quantiser noise has uniform distribution from *-q/2* to *q/2*
- SNR for quantisation to *M* bits is (Sec 7.5.2):

$$\mathrm{SNR} = 1.76 + 6M \ (\mathrm{dB})$$



Noise Reduction Thru Filter

- FIR Filter operation:

$$y_q(n) = x_q(n) * h(n)$$
$$= x(n) * h(n) + e_q(n) * h(n)$$

- Output noise is $e_q(n) * h(n)$
- Input/output noise variances related as:

$$\sigma_v^2 = \sigma_e^2 \sum h^2(k)$$

- For example, consider:

$$y(n) = \frac{1}{N}\left[\sum_{i=0}^{N-1} x(n-i)\right]$$

- Output noise variance is given by:

$$\sigma_v^2 = \sigma_e^2 \sum_{i=0}^{N-1} 1/N^2 = \sigma_e^2/N$$

- So thermal and/or quantisation noise is reduced in power by a factor of *N*

Coefficient Quantisation Errors

- Consider 2nd order FIR response:

$$H(z) = (1 - r\exp(j\theta)z^{-1})(1 - r\exp(j\theta)z^{-1})$$
$$= (1 - 2r\cos(\theta)z^{-1} + r^2 z^{-2})$$

- The terms $2r\cos(\theta)$ and r^2 can only take on certain values
- Zeros limited to grid of points

Locations of zeros with 4 bit arithmetic:

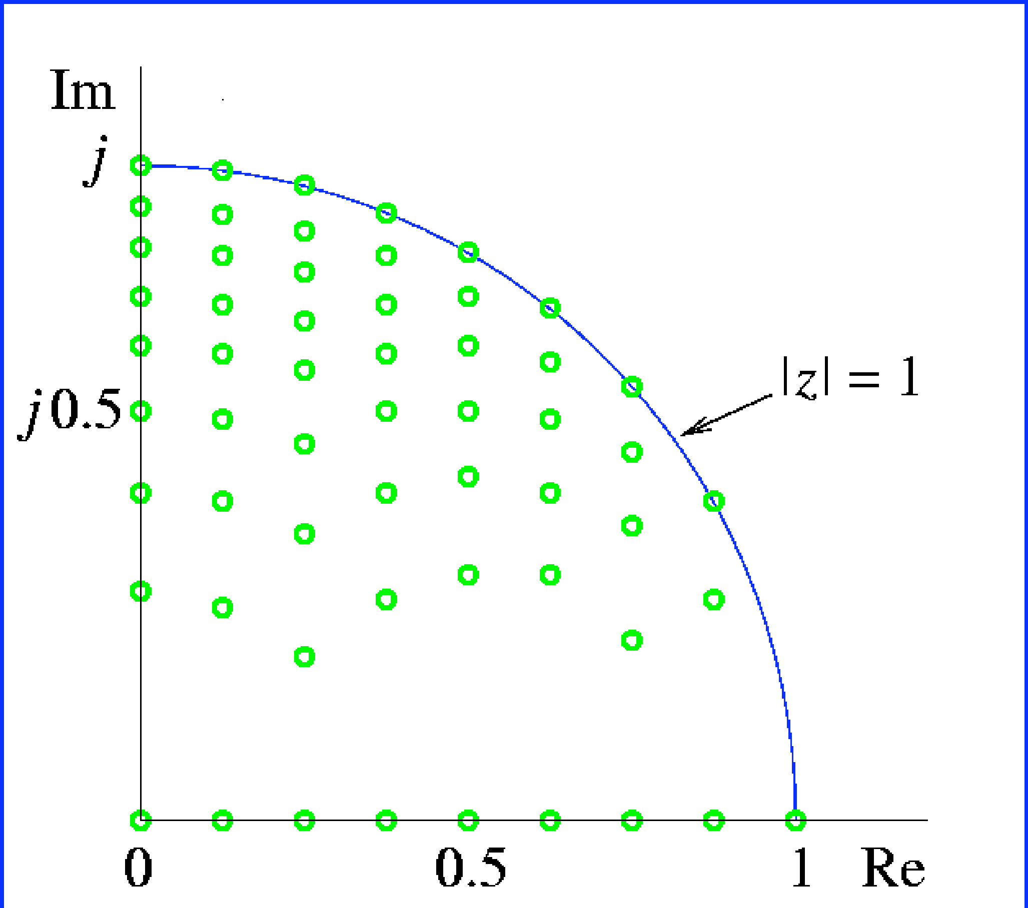
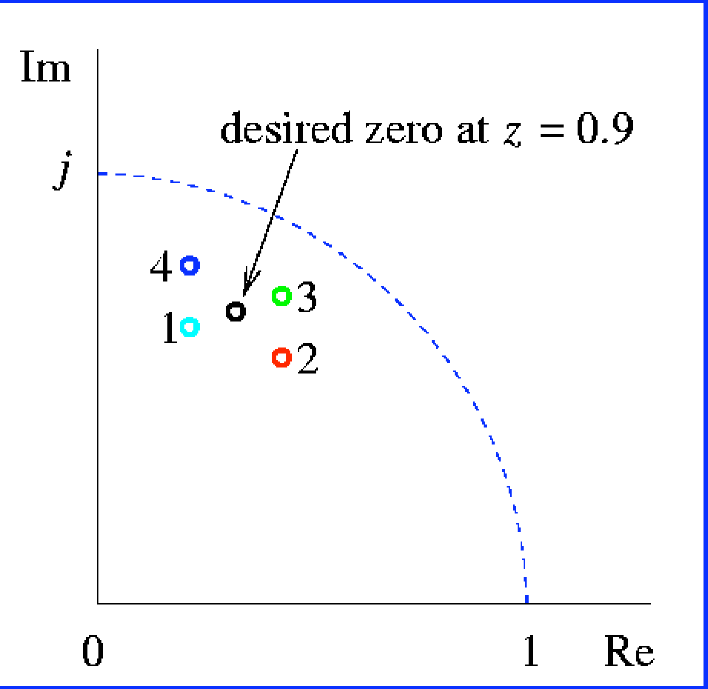


Fig 6.13

Location of Zero:



Frequency Response:

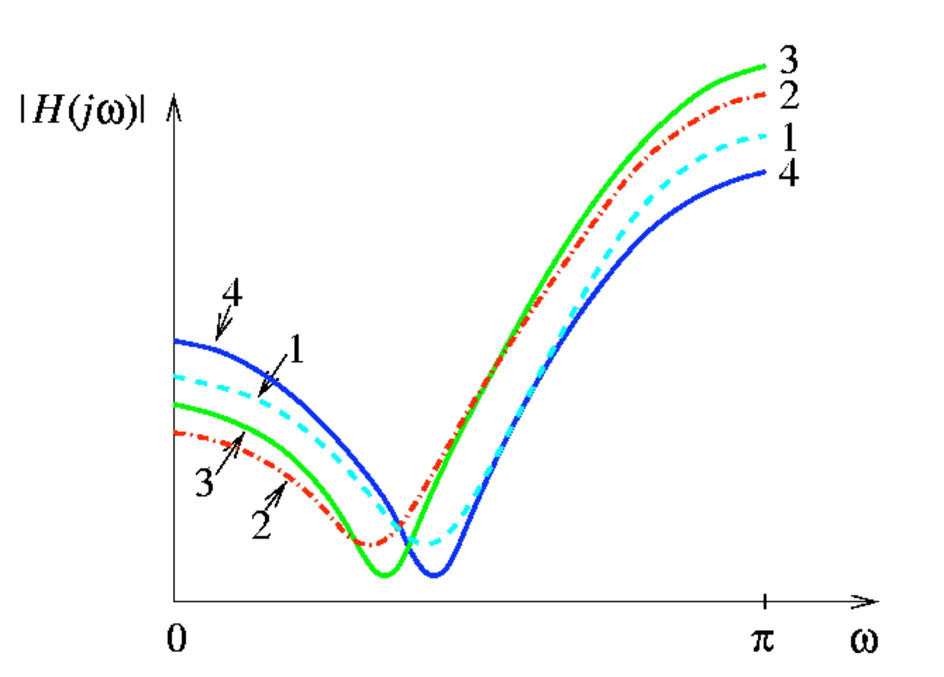


Fig 6.14

- Formulas exist to estimate No of bits b_c for given filter of length N taps:

$$b_c = 1 + \log_2 \left[\frac{\Delta t (f_1 + f_2)}{\delta_m - \delta_0} \sqrt{\frac{N}{3}} \right]$$

- f_1, f_2 : passband/stopband edge
- δ_0,δ_m: ripple before/after quantisation

FIR Filter Applications

- Matched filter
- Consider a general signal *f(t)*
- If it is corrupted by white Gaussian noise:
  - Optimum detector is the Matched Filter
  - It maximises the signal in the output *g(t)*

- Matched filter defined in frequency as:

$$H(\omega) = kF^{*}(\omega)\exp(-j\omega t_m)$$

- Frequency response is *matched*

- The impulse response is:

$$h(t) = kf^{*}(t_m - t)$$

- Matched Filter is *time-reverse* of *f(t)* with time delay t_m

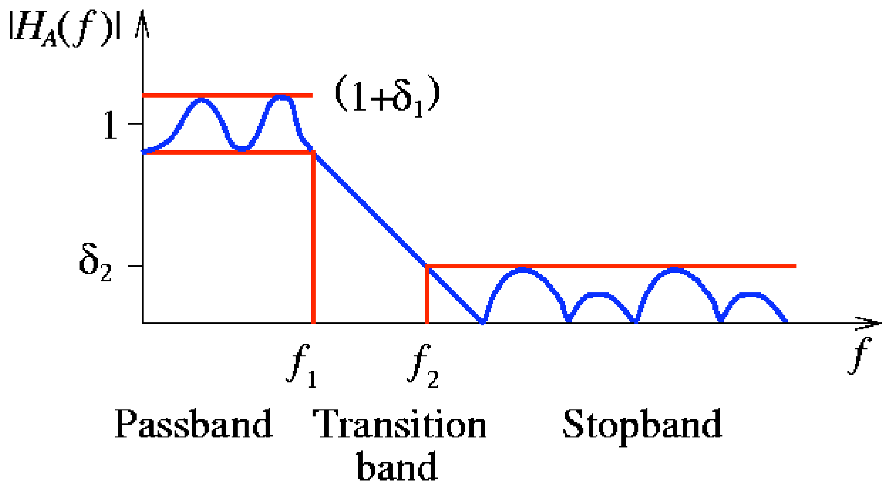
Other Receiver Designs

- The Wiener filter: $$H(z) = \frac{F^*(z)}{F(z)F^*(z) + n_0^2}$$
- If noise n_0^2 >> F(z)F*(z): $$H(z) = \frac{F^*(z)}{n_0^2}$$
- If noise level is low: $$H(z) = \frac{1}{F(z)}$$

Chapter Six Summary

- Design of Linear Phase FIR Filters
- Use of Windows to improve stopband attenuation
- Implementation issues and finite precision effects

Window Characteristics (Slide 16)



passband ripple = $20 \log_{10}(1+\delta_1)$ dB

stopband rejection = $20 \log_{10}(\delta_2)$ dB

transition band $\Delta f = f_1 - f_2$ Hz

Fig 6.10