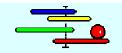
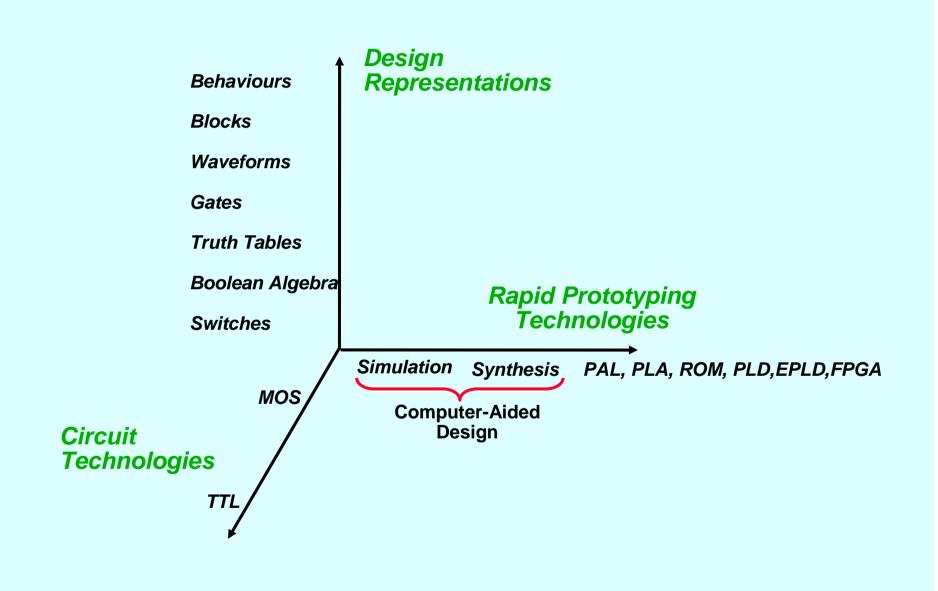


Review for B33DV2-Digital Design

The Elements of Modern Digital Design

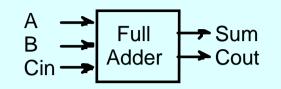




Combinational vs. Sequential Logic

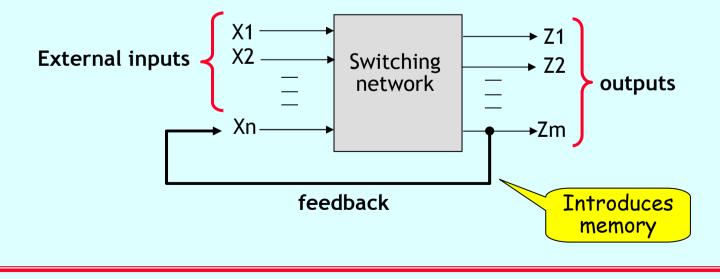
• Combinational logic

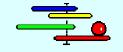
- → no feedback among inputs and outputs
- \rightarrow outputs are a pure function of the inputs
- → e.g., full adder circuit:
- → (A, B, Carry In) mapped into (Sum, Carry Out)



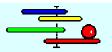
• Sequential logic

→ Network implemented from switching elements or logic gates. The presence of feedback distinguishes between sequential and combinational networks





Sequential Systems



• Sequential logic

- → network typically has only a limited number of unique configurations
 - > these are called states
 - e.g., a simple traffic light controller sequences infinitely through four states
- → outputs depend on inputs and the entire history of execution!
 - \succ output and new state is a function of the inputs and the old state
 - outputs = F(inputs, present_state)
- → includes components to remember the current state
 - > i.e., the fed back inputs are the state

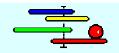
• Synchronous systems

- → internal state changes synchronized to edges of a **clock** signal
- → effect of changes propagate between clock edges,
 - \succ as in a standard Intel PC



Propagation time

Representations used in Digital Design



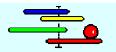
- Boolean algebra
 - → Mathematical representation of static binary logic
- Truth tables
 - → Tabular representation of ALL input/output combinations
- Minterms, (maxterms)
 - → Boolean algebra representations of truth tables
- Logic diagrams
 - → Schematic diagram of logic system
- Timing diagrams
 - → Shows timing relationships between system units
- Karnaugh maps
 - → Geometrical representation of truth table data
- State transition diagrams
 - → For sequential systems shows relationship between the states of a system
- ASM charts
 - → For sequential systems shows sequencing between the states of a system

Rules of Boolean Algebra

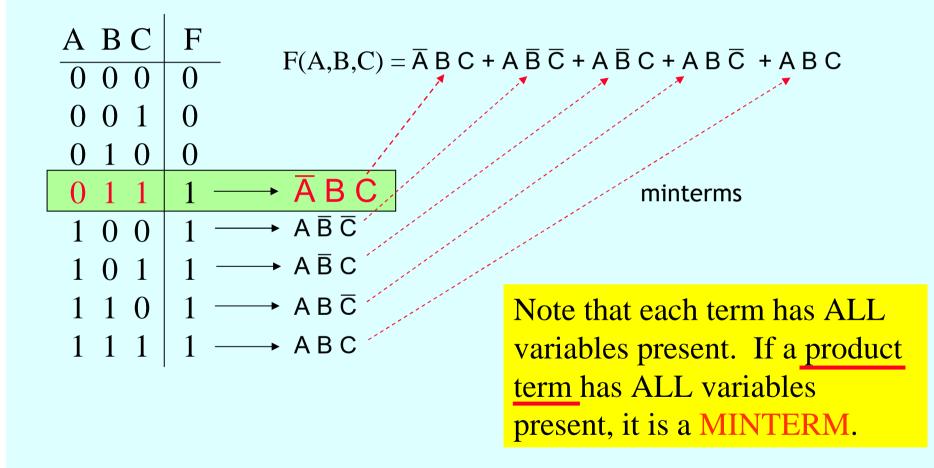
- **1. Commutative Law** \rightarrow A . B = B . A
 - → A + B = B + A
- 2. Associate Law
 - → (A . B) . C = A . (B . C)
 - $\Rightarrow (A + B) + C = A + (B + C)$
- 3. Distributive Law
 → (A + B) . C = (A . C) + (B . C)
 - $\Rightarrow (A \cdot B) + C = (A + C) \cdot (B + C)$
- 4. Identities
 - → A + 0 = A
 - → A.1=A
- 5.
 - → A + 1 = 1
 - → A.0=0

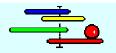
- 6. $\rightarrow A + A = A$
 - → A . A = A
- 7.
 - → A + (A') = 1
 - → A . (A') = 0
- 8. Inverse
 - → (A')' = A
 - 9. De Morgan's Theorem
 - → (A+ B)' = (A') . (B')
 - → $(A \cdot B)' = (A') + (B')$
 - Evaluation order
 - → Parentheses
 - → ' (NOT)
 - → * (AND)
 - → + (OR)

Truth Table to SOP Form



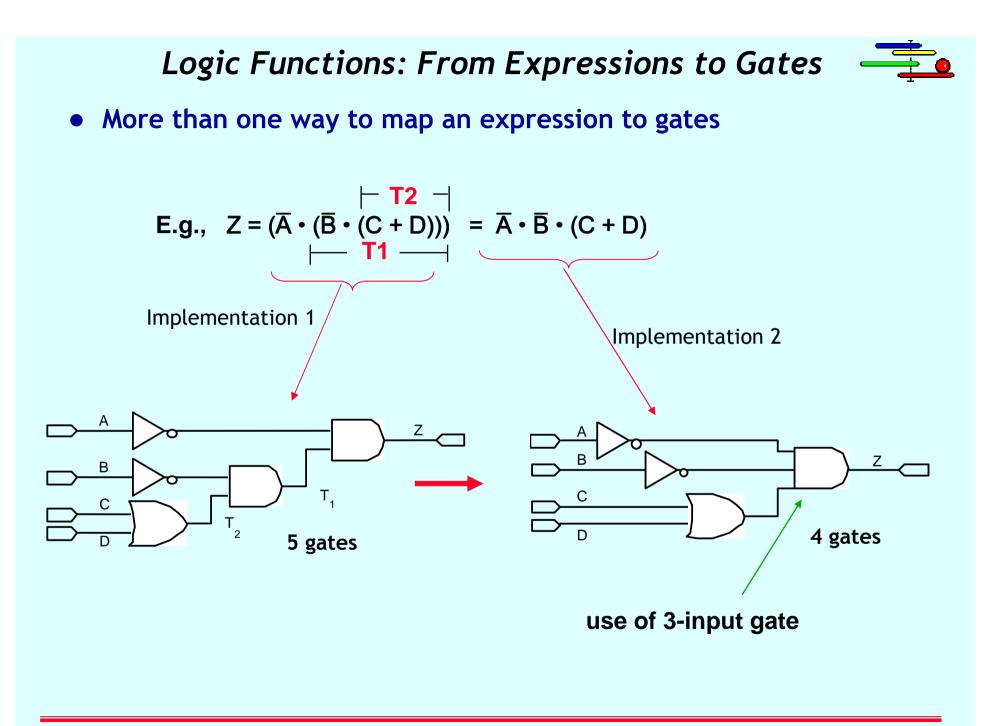
• Sum-Of-Products (SOP) form can be written directly from truth table.

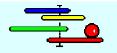




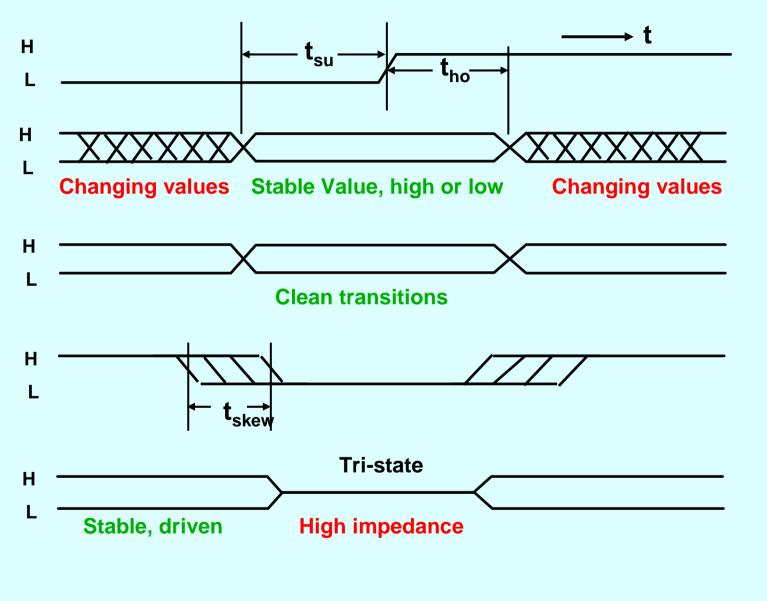
• Each line in a truth table represents a Minterm.

Row No.	A B C	Minterms	
0	0 0 0	$\overline{A} \overline{B} \overline{C} = m_0$	
1	0 0 1	$\overline{A} \overline{B} C = m_1$	
2	0 1 0	$\overline{A} B \overline{C} = m_2$	
3	0 1 1	$\overline{A} B C = m_3$	
4	1 0 0	$A \overline{B} \overline{C} = m_4$	
5	1 0 1	$A \overline{B} C = m_5$	
6	1 1 0	$A B \overline{C} = m_6$	
7	1 1 1	$ABC = m_7$	

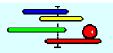




Timing Diagram Notation

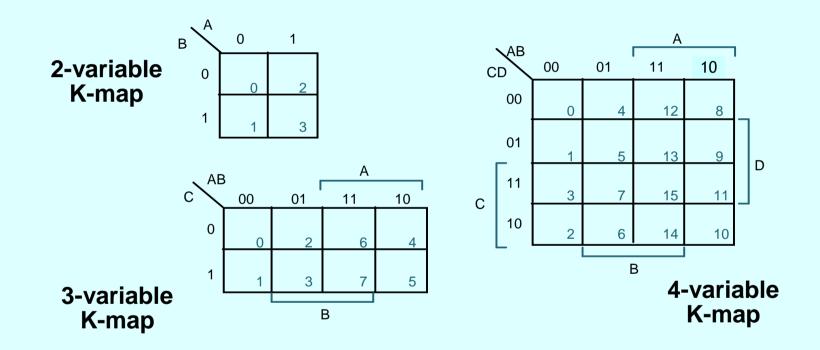


Karnaugh Map



• Karnaugh Map Method

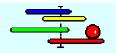
→ K-map is an alternative method of representing the truth table that helps visualize adjacent terms in up to 6 dimensions



→ Numbering Scheme: 00, 01, 11, 10

→ Gray Code -- only a single bit changes one code word to the next

Simplification using K-Maps

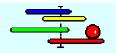


- Grouping blocks of '1'
 - → A group must consist of <u>16,8,4,2 or 1 cells</u>
 - → Each cell must be horizontally and/or vertically adjacent to cells in the other group
 - → Always include the largest number of '1's in a group
 - → Each '1' in the map should be included in a group
 - → Groups can overlap
 - → Map edge cells connect in a loop to cells at the opposite edge

• Naming groups

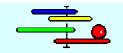
- → The product description for a group will include ALL variables that are CONSTANT over the group
- → For example, for a 4-variable map
 - > An 8-cell group is described by a 1-variable product term
 - > An 4-cell group is described by a 2-variable product term
 - > An 2-cell group is described by a 3-variable product term
 - > An 1-cell group is described by a 4-variable product term

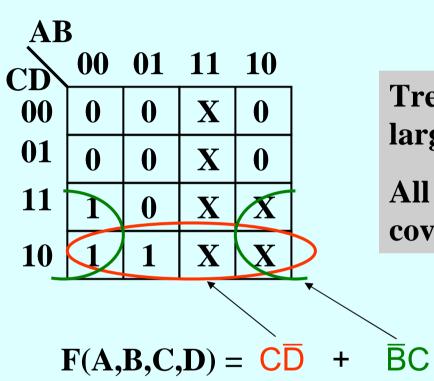
Minimal Solution



- A minimal SOP will consist of prime implicants.
- A minimal SOP equation will have all of the essential prime implicants on the map. By definition, these cover a minterm that may not be covered by some other prime implicant.
- The minimal SOP equation may or may not include nonessential prime implicants. It will include non-essential prime implicants if there are '1's remaining that have not been covered by an essential prime implicant.

Don't Cares treated as '0's or '1's

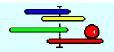




Treat X's as 1's to make larger groupings.

All X's do not have to be covered.

Variable-Entered Karnaugh Maps (VEM)

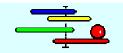


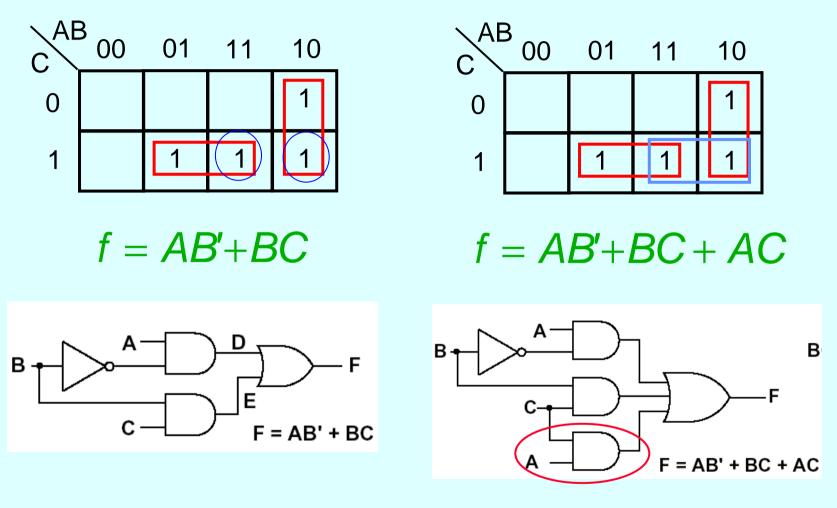
• VEM Key idea:

- → Represent values of function in terms of its variables (called mapentered variables) within Karnaugh map framework
- → Group like variables in Karnaugh map cells

- → Minimisation approach
 - > Map all 1's
 - > Convert 1's to X's (i.e. don't cares)
 - > Map SIMILAR components

Hazards in Combinational Circuits

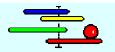




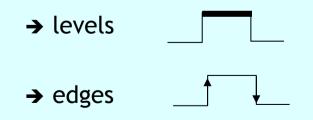
To avoid hazards:

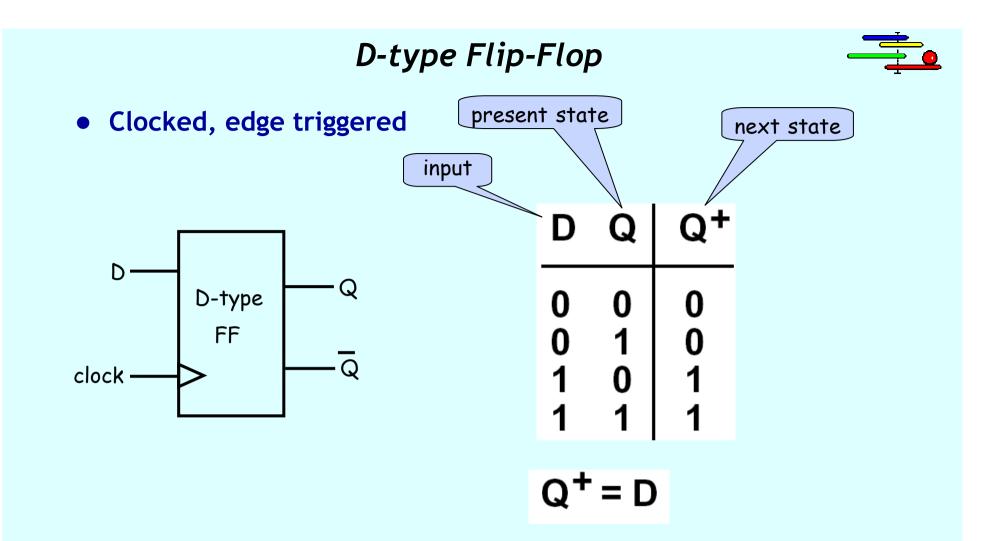
every pair of adjacent 1s should be covered by a 1-group

Flip-Flops/ Latches

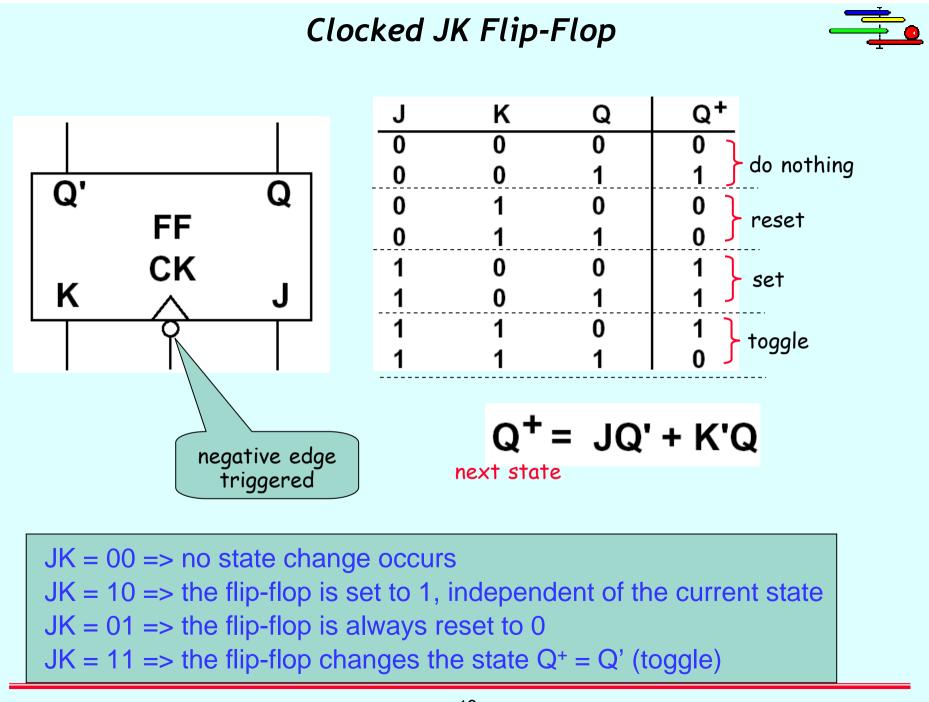


- Latches and Flip-Flops are devices that can have two internal states (0,1)
- The output of a latch or a Flip-Flop (FF) is dependent upon its
 CURRENT STATE
 - → CURRENT INPUTS.
- Latches and FFs are the simplest examples of sequential systems.
- State transitions based on

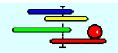




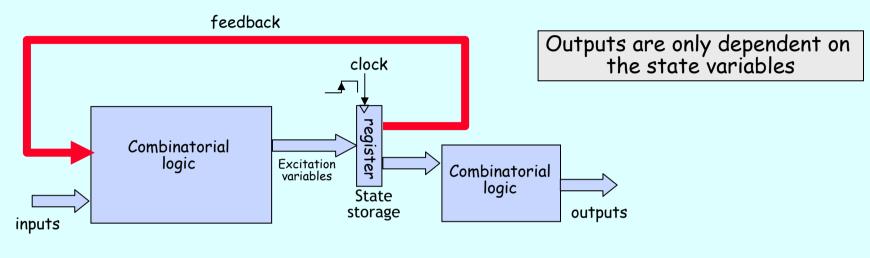
The next state in response to the rising edge of the clock is equal to the D input before the rising edge



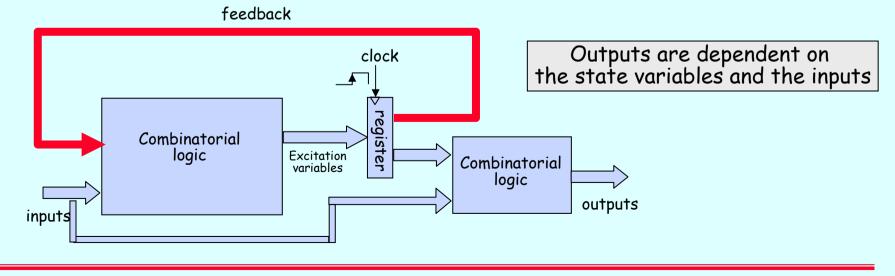
Sequential System Concepts



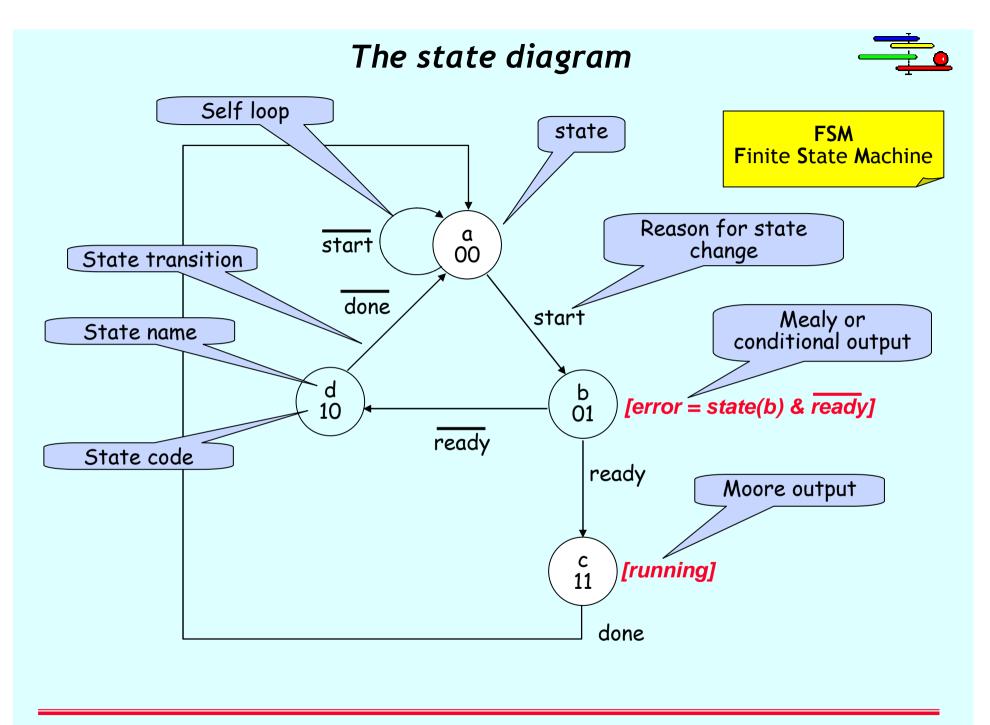
• Moore machine



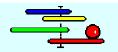
• Mealy machine

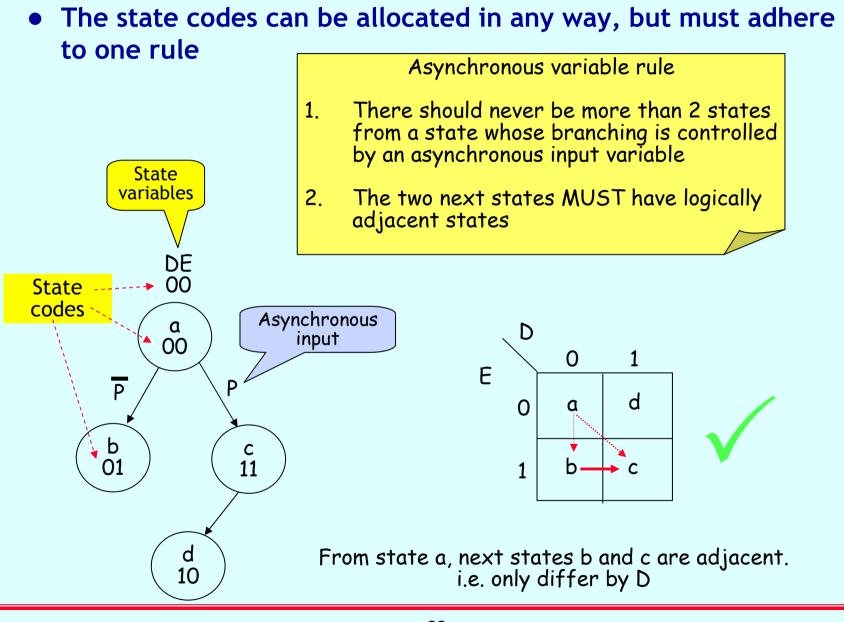


20



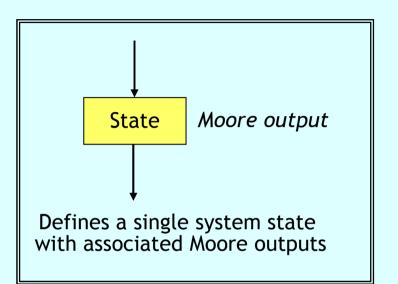
Assignment of state codes

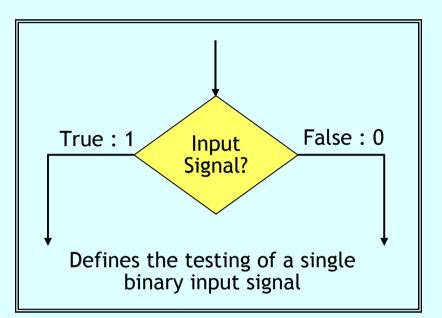


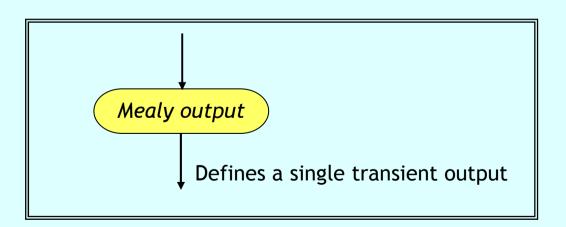


Algorithmic State machine (ASM) chart notation —

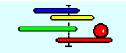
• Similar to flow charts



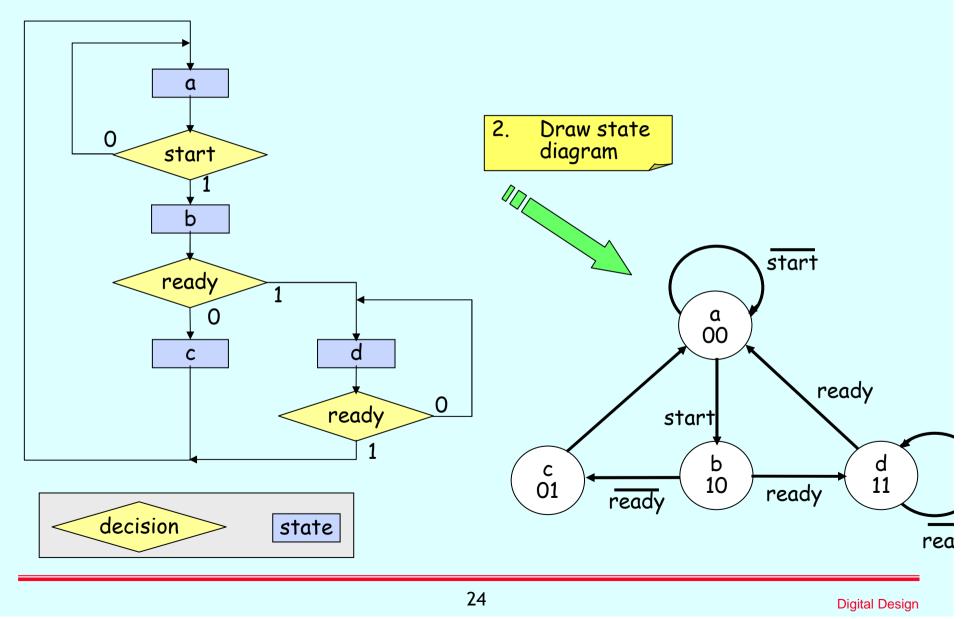




Algorithmic State machine charts



• Based on flow charts



Excitation table for D-type excitation equations

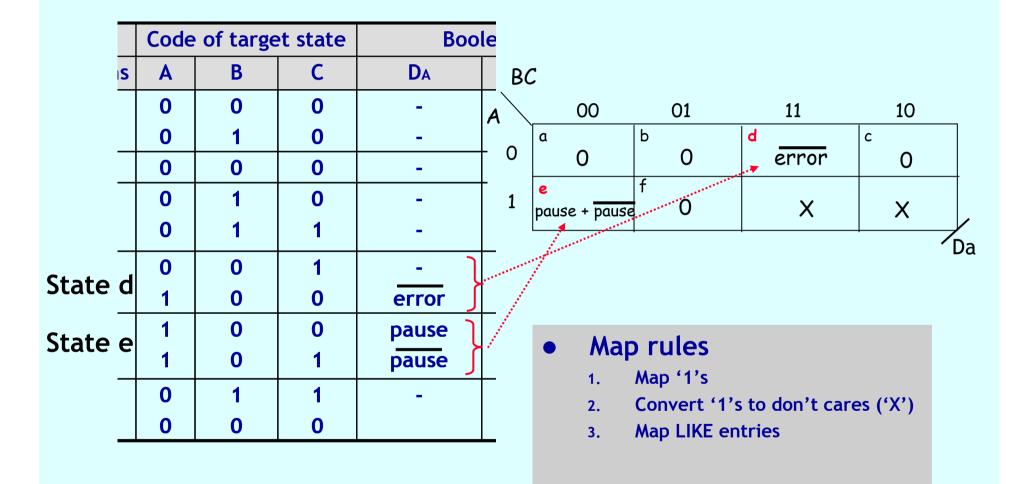
• Table of state transitions

		Code of target state			Boolean on transition		
state	transitions	Α	В	C	DA	Dв	Dc
a	a→a	0	0	0	-	-	-
	a-) c	0	1	0	-	start	-
b	b→a	0	0	0	-	-	-
С	c→c	0	(1)	0	<u>-</u>	-+ wait	-
	c→d	0	1	1	-	wait	wait
d	d→b	0	0	1	-	-	error
	d→e	1	0	0	error	-	-
е	e→e	1		0	pause	-	-
	e→f		0	1	pause	-	pause
f	f→d	0	1	1		done	done
	a→a	0	0	0			
					$\widehat{1}$		

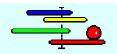
D-input excitation values for state variable 'A' D-type flipflop

Excitation table for D-type excitation equations

• Transitions table to Karnaugh Map

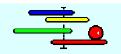


System design



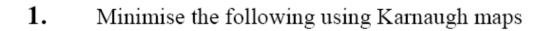
Digital system consist of two cooperating units → Data → Control Data in Data control Control signals Control signals manipulation Data out \rightarrow E.g. A computer, a graphics card, an ethernet interface, etc

Data manipulation components



- The components available to the designer include
 - → Registers
 - → Counters
 - > Up, down
 - → Shift registers
 - ➤ Left, right
 - ➔ Multplexors
 - > Also called data selectors
 - ➔ De-multiplexors
 - → Comparators
 - ➔ Arithmetic units
 - > Adders, subtractors, incrementors, multipliers, ALUs
- At design time it is important to see these blocks as **BLACK BOXES**
 - → Their individual design can be decided at a later stage
 - → Need only define inputs and outputs
 - Direction
 - > Polarity
 - Level or edge

Digital Design : Tutorial



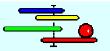
a.
$$F(a,b,c) = \sum m(0,1,3,4,6)$$

b.
$$F(a,b,c,d) = \sum m(2,3,7,9,11,12,15)$$

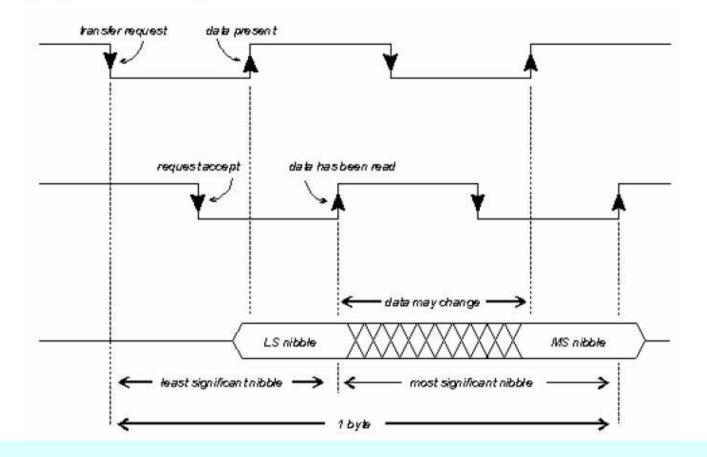
c. $F(a,b,c,d,e) = \sum m(5,6,7,9,11,17,19,21,25,26,27,30,31)$

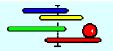
1.
a.
$$F = \overline{AB} + \overline{BC} + \overline{AC} + \overline{AC}$$

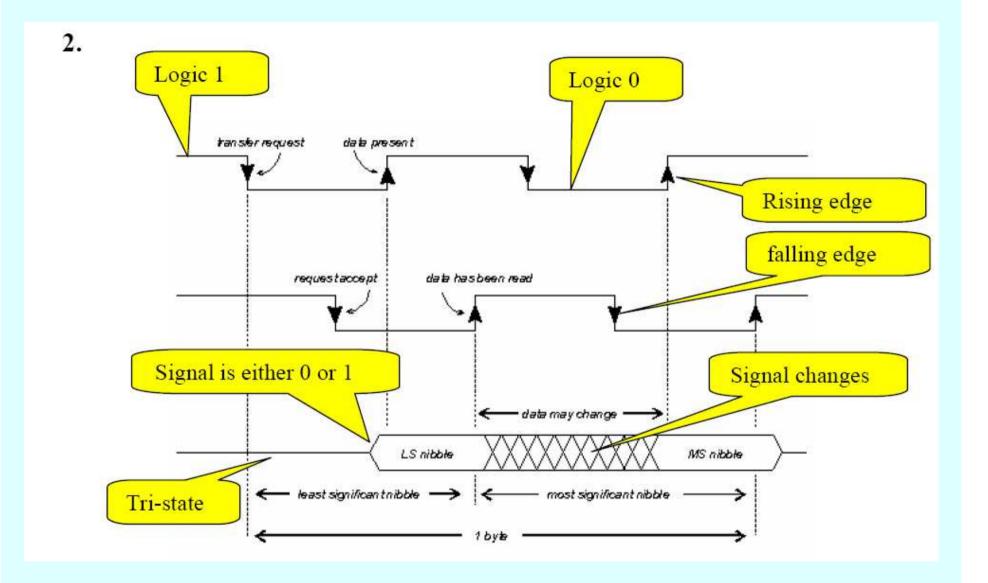
b. $F = CD + \overline{ABC} + \overline{ABD} + \overline{ABCD}$
c. $F = \overline{BCE} + \overline{ABD} + \overline{BCDE} + \overline{ABCD} + \overline{ABCE}$

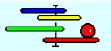


2. Describe the different graphical aspects of this waveform diagram. Ignore the purpose of the diagram.







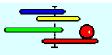


3. Convert the following 4 variable function to a standard Karnaugh map.

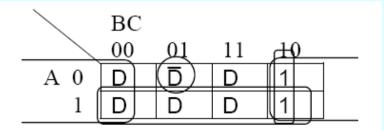
 $F(a,b,c,d) = \sum m(1,2,4,5,7,9,11,12,13,15)$

Convert this 4 variable Karnaugh map to a 3 variable map entered Karnaugh map with \mathbf{d} as the map entered variable.

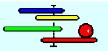
Truth table ::



А	В	С	D	Output	Map entry	
0	0	0	0	0	D	
0	0	0	1	1		
0	0	1	0	1	D	
0	0	1	1	0	D	
0	1	0	0	1	1	
0	1	0	1	1		
0	1	1	0	0	D	
0	1	1	1	1	D	
1	0	0	0	0	D	
1	0	0	1	1	D	
1	0	1	0	0	D	
1	0	1	1	1	D	
1	1	0	0	1	- 1	
1	1	0	1	1	I	
1	1	1	0	0	D	
1	1	1	1	1	D	



 $\mathbf{F} = \mathsf{B}\overline{\mathsf{C}} + \mathsf{A}\mathsf{D} + \mathsf{B}\mathsf{D} + \overline{\mathsf{C}}\mathsf{D} + \overline{\mathsf{A}}\overline{\mathsf{B}}\mathsf{C}\overline{\mathsf{D}}$

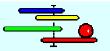


4. Answer the following questions

- **a.** Show how a glitch pulse can be created by a race condition setup by a single input signal.
- **b.** Describe the operation of a D-type flip-flop.
- **c.** Draw the structure of a Moore state machine.
- **d.** Outline the problem of state consistency.
- **e.** State the "asynchronous rule" for state assignment and indicate why it is important when dealing with asynchronous input signals.

4.

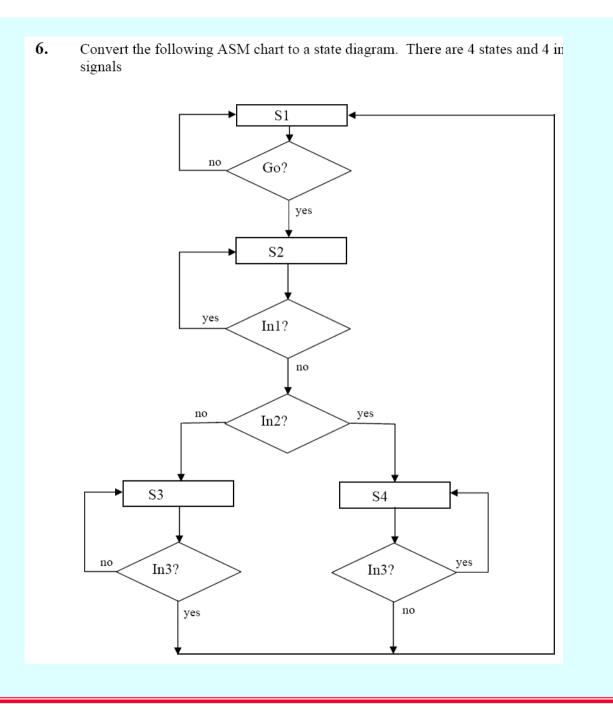
These questions are bookwork. Refer to the notes.

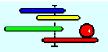


 What are the rules to produce a mapping from a Variable Entered Karnaugh Map (VEM).

5.

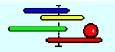
- a. map '1's
- b. convert '1's to don't cares
- c. map squares with identical contents as per a normal karnaugh map.



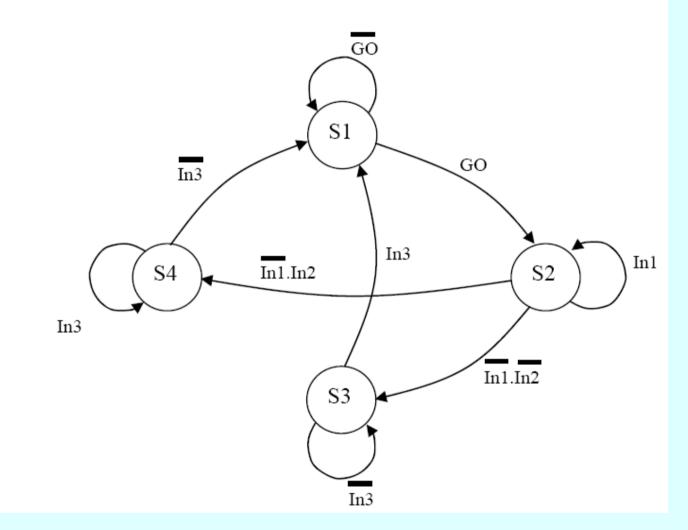


List all transitions ::

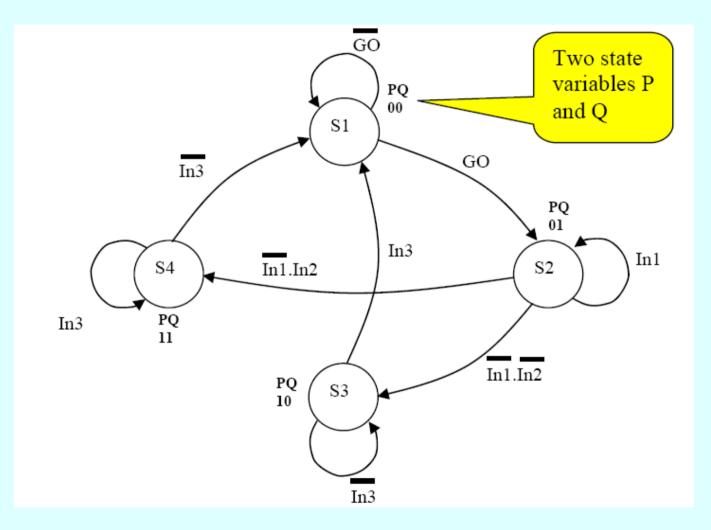
State	Transition	Logic of transition
S1	$s_1 \rightarrow s_1$	GO
	$s_1 \rightarrow s_2$	GO
	$s_2 \rightarrow s_2$	In1
S2	$s_2 \rightarrow s_3$	In1.In2
	$s_2 \rightarrow s_4$	In1.In2
S3	S3 → S3	In3
55	$s_3 \rightarrow s_1$	In3
S4	$S4 \rightarrow S4$	In3
	$S4 \rightarrow S1$	In3

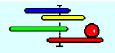


Resulting state machine.

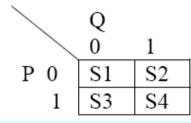


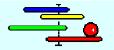
7. Convert the state diagram of question 6 to a set of excitation equations for a D-type implementation of the state machine.

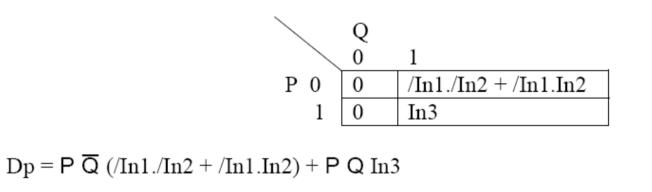


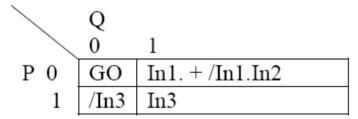


State	Transition	Target state		Dp	Dq
		P	Q		
S1	S1→S1	0	0		
	S1→S2	0	1		GO
S2	S2→S2	0	1		In1
	S2→S3	1	0	/In1./In2	
	S2→S4	1	1	/In1.In2	/In1.In2
S3	S3→S3	1	0	/In3	
	S3→S1	0	0		
S4	S4→S4	1	1	In3	In3
	S4→S1	0	0		



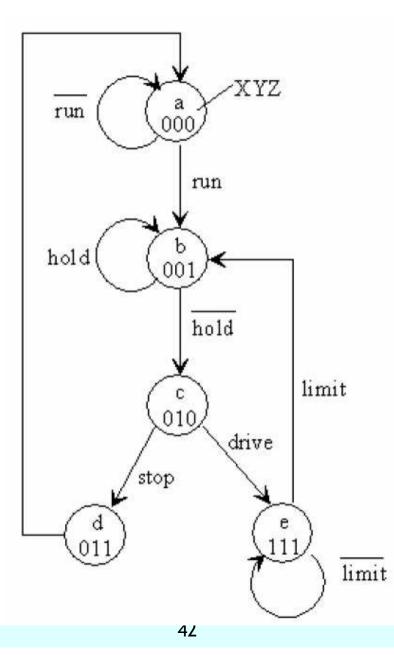


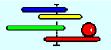




 $Dq = P Q GO + \overline{P} Q (In1 + /In1.In2) + \overline{Q} P /In3 + P Q In3$

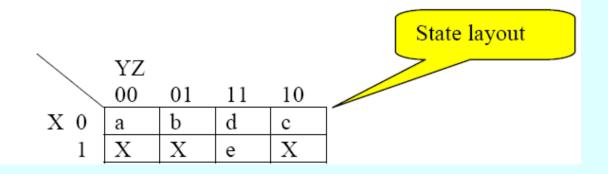
8. Convert the following state diagram to a set of D-type excitation equations.

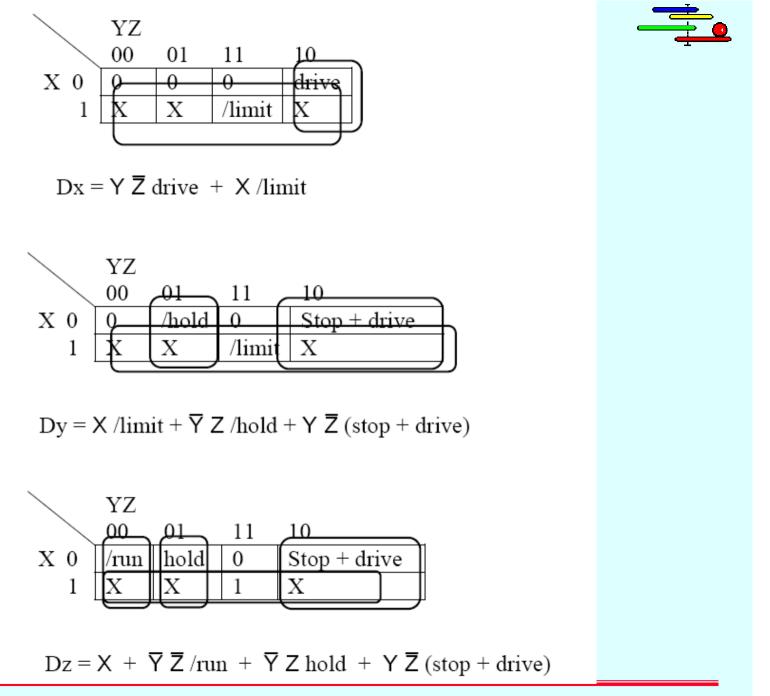


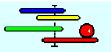


8.

State	Transition	Target state					
		Χ	Y	Ζ	Dx	Dy	Dz
a	a→a	0	0	0			
	a→b	0	0	1			/run
b	b→b	0	0	1			hold
	b→c	0	1	0		/hold	
c	c→d	0	1	1		stop	stop
	c→e	1	1	1	drive	drive	drive
d	d→a	0	0	0			
e	e→e	1	1	1	/limit	/limit	/limit
	e→b	0	0	1			limit

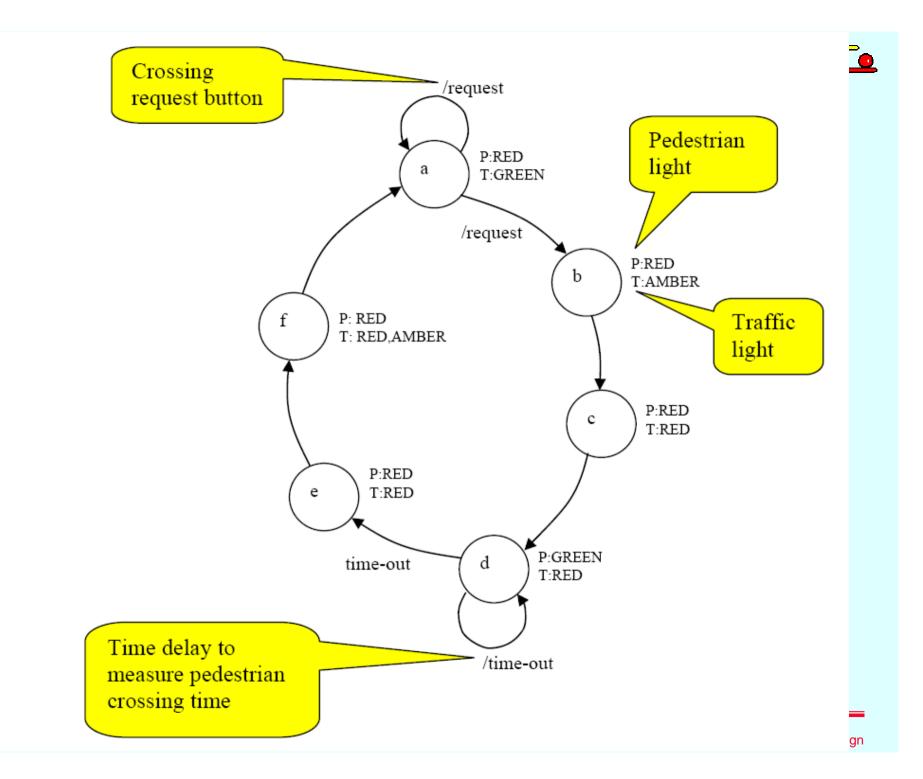


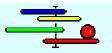




ROAD G Traffic light Button G Pedestrian light

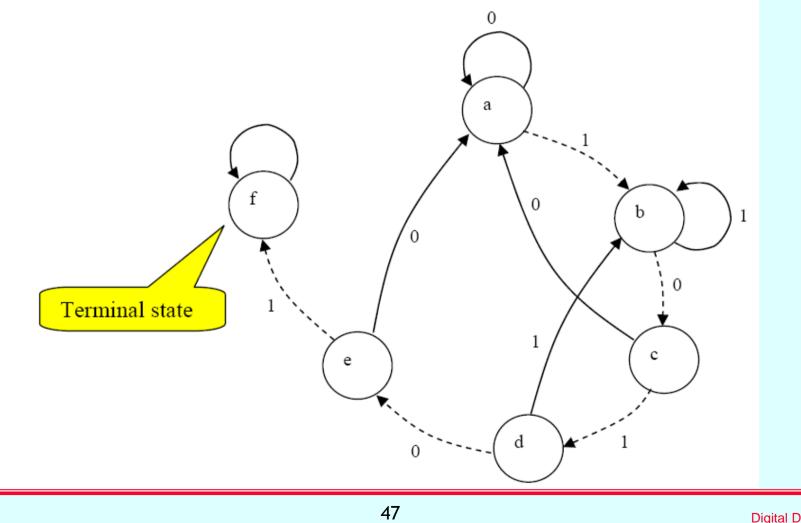
9. Design a state machine to control a simple. "pedestrian crossing".





10. Design a state machine to detect the value '10101' in a serial stream of data.

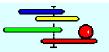
10101 pattern detector state diagram. Dotted line shows main detection route.



11. List the steps requires to solve a state machine problem that starts with a specification and finishes with a D-type flip-flop implementation.

11.

- a. ASM chart
- b. State diagram
- c. Transition table
- d. State location karnaugh map
- e. Karnaugh map and associated boolean excitation equation for each state variable
- f. Karnaugh maps and associated boolean equation for each output

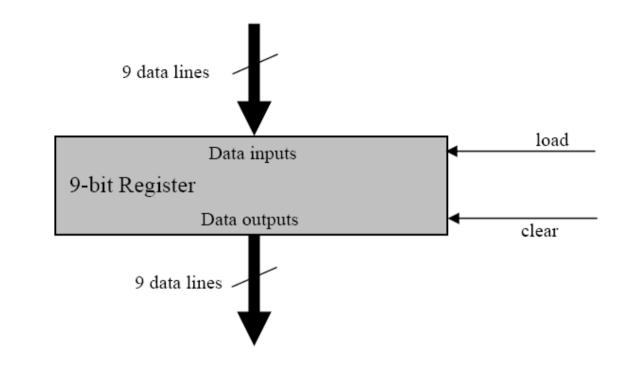


12. List a general set of data components that can be used for digital system design.

12.

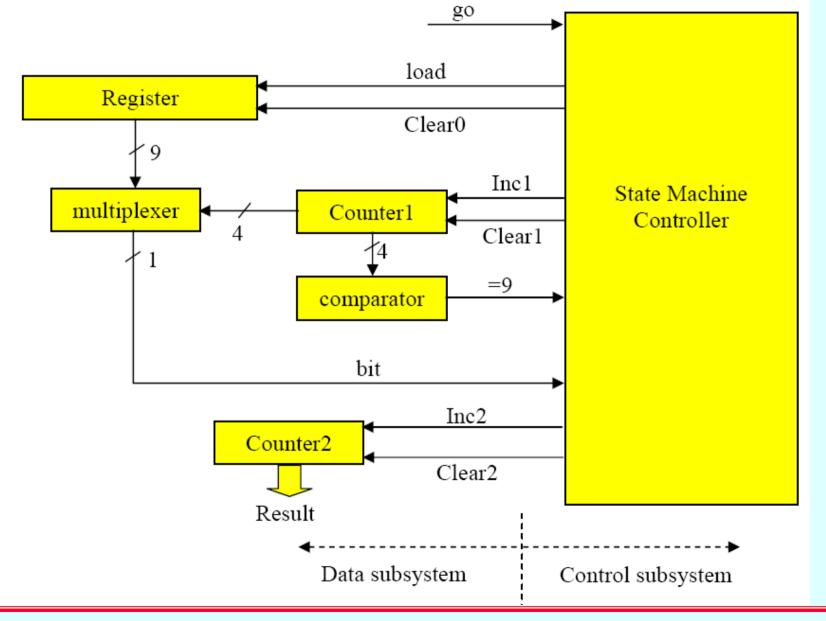
a.	registers
b.	counters
c.	shift registers
d.	multplexers
e.	de-multiplexers
f.	arithmetic units
g.	comparators

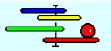
13. Design a state machine based system to count the number of '1's in a register of length 9 bits. The starting point for your design is as follows

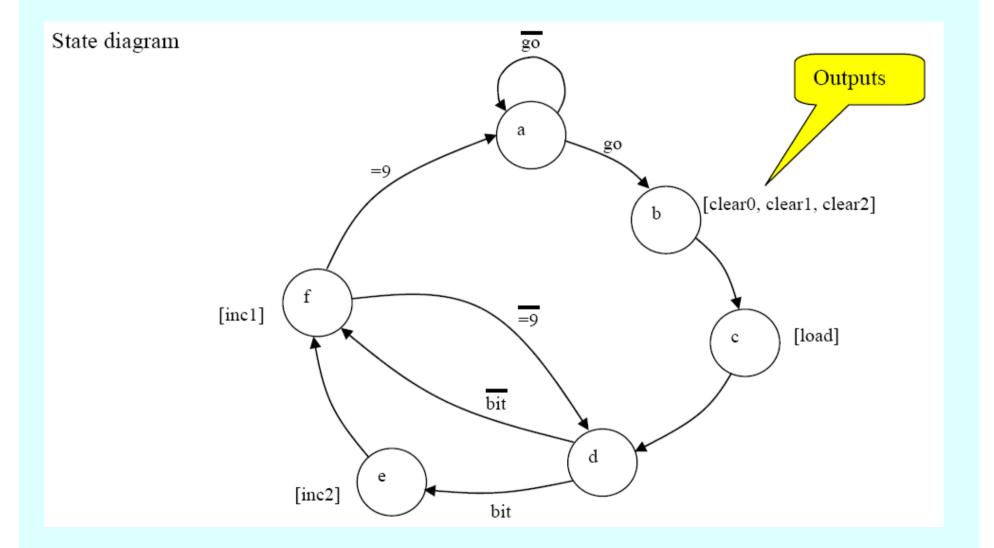


There are two control inputs - load and clear

Block diagram structure







14. (Exam question from 2003)

A manufacturer of vacuum cleaners wishes to experiment with robotic cleaners. They plan to design a cheap model that has little intelligence that bumps its way around a room in an almost random manner. It is proposed to build a series of test versions to evaluate the concept.

This initial version has the following hardware features

- A start button
- Motors can drive forward, backwards, turn left, turn right, and stop
- Three bump sensors, front left, front right, and rear provide detection of the environment
- An on-board timer, when triggered, gives a signal after 2 seconds

The operational requirements are

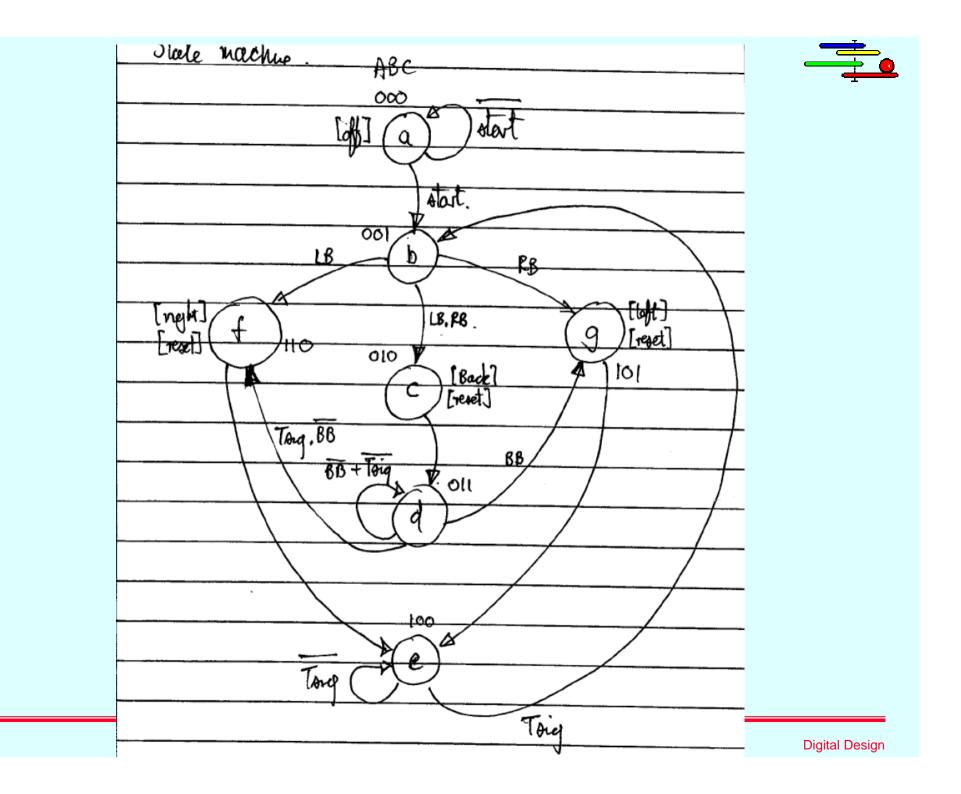
- On start drive forward.
- If left bump activated , turn right for 2 seconds then drive forward.
- If right bump activated, turn left for 2 seconds then drive forward.
- If both left and right bump activated, reverse for 4 seconds, turn right for 2 seconds, then drive forward.
- If rear bump, drive forward for 2 seconds, turn left for 2 seconds, then drive forward.

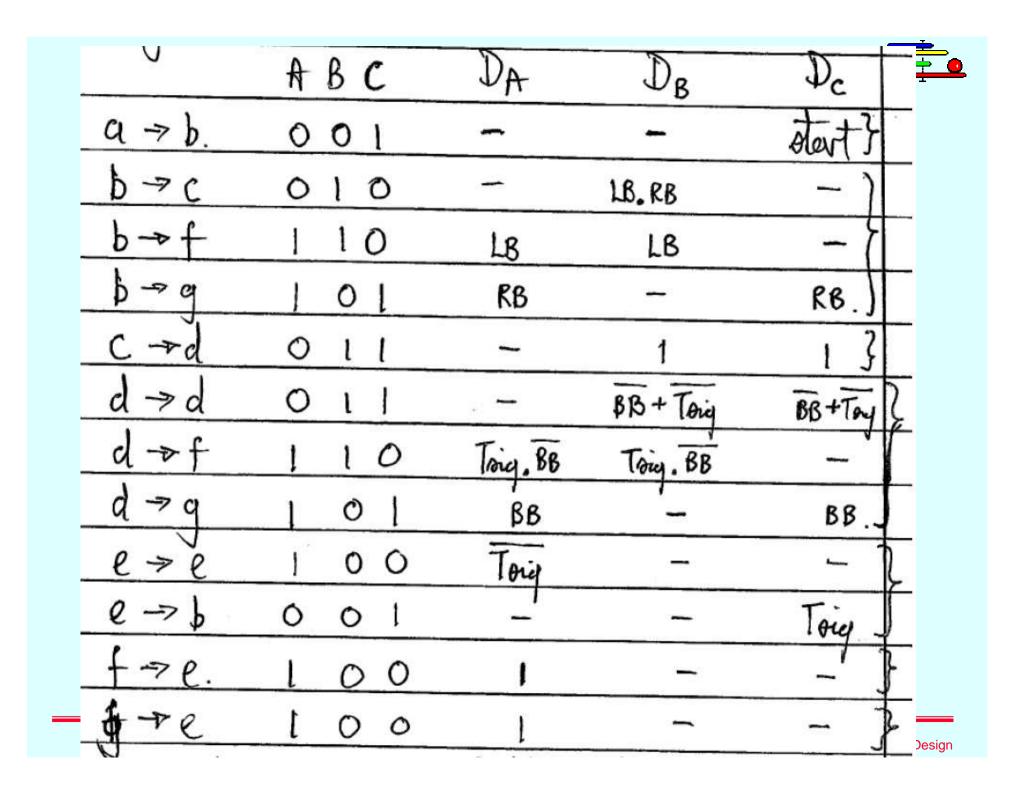
The control system is to be built as a state diagram.

- a) Draw a system block diagram showing the various subsystems and the relevant signals.
- b) Create a state diagram for the vehicle controller.
- c) From the state diagram derive a set of excitation equations for a D-type flip-flop implementation.

d) Suggest changes to the hardware and state diagram to allow the following modification

- If both left and right bump activated, reverse for 2 seconds, increment a 3-bit counter.
 - If counter is less than 4 turn right for 2 seconds, then drive forward
 - If counter is greater than 4 turn left for 2 seconds, then drive forward
 - (note : when counter is at 7 the next count sets count to zero)

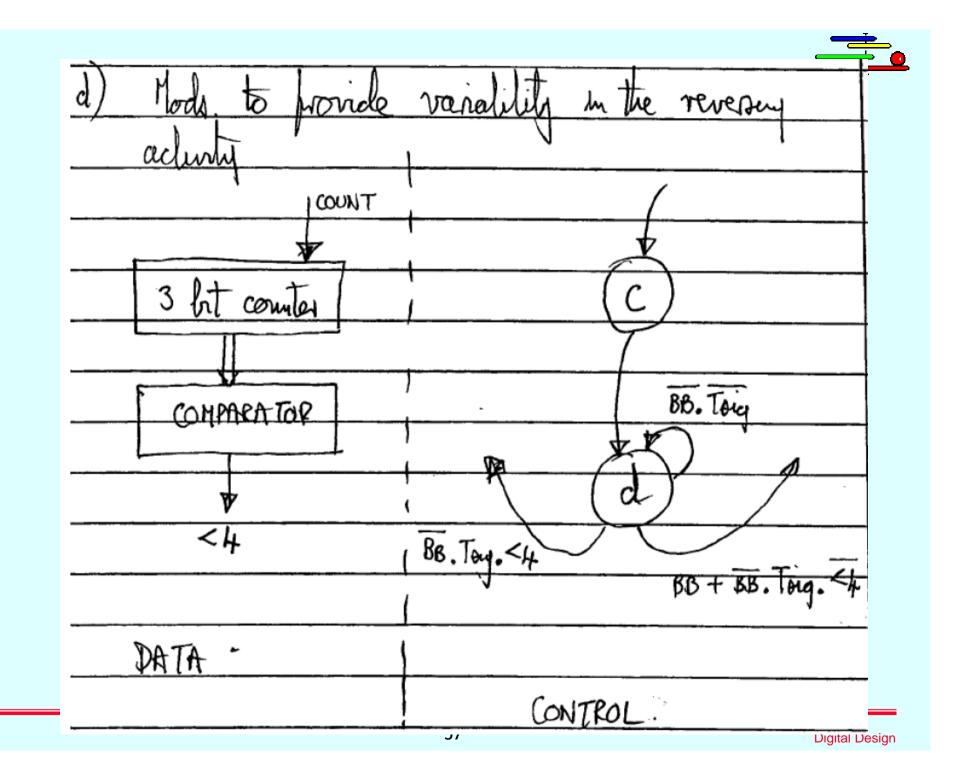




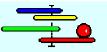
$$D_{A} = A, B + B, C + A, C + A, T_{Bry} + C, (LB + RB)$$

$$D_{B} = \overline{ABC} + \overline{ABC}, LB + \overline{ABC} (\overline{BB} + T_{Bry})$$

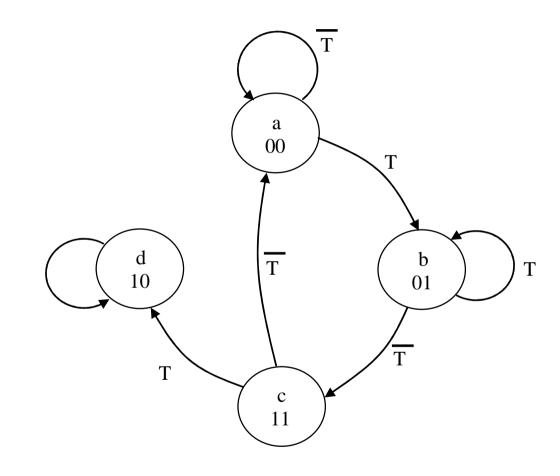
$$D_{C} = \overline{AB} + \overline{AC} \overline{Dat} + \overline{AC}, RB + \overline{ABC}, T_{Bry}$$



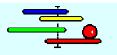
2006 Q1



Convert the following state diagram into a set of excitation equations for a D-type implementation. T is a single input variable and each of the 4 states (a to d) has been given an appropriate binary coding.

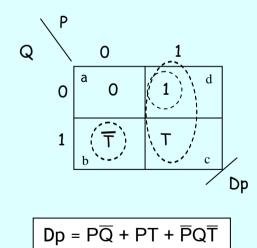


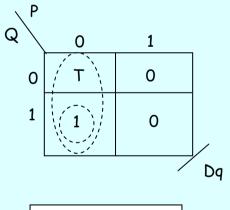
State transition table



State	Transitions	Code of target state		Boolean on transition		
		Р	Q	Dp	Dq	
а	a→a a→b	0	0	-	-	
	a→b	0	1	-	Т	
b	b→b b→c	0	1	_	T	
	b→c	1	1	T	T	
с	c→a c→d	0	0	-	-	
	c→d	1	0	Т	-	
d	d→d	1	0	1	-	

Karnaugh maps

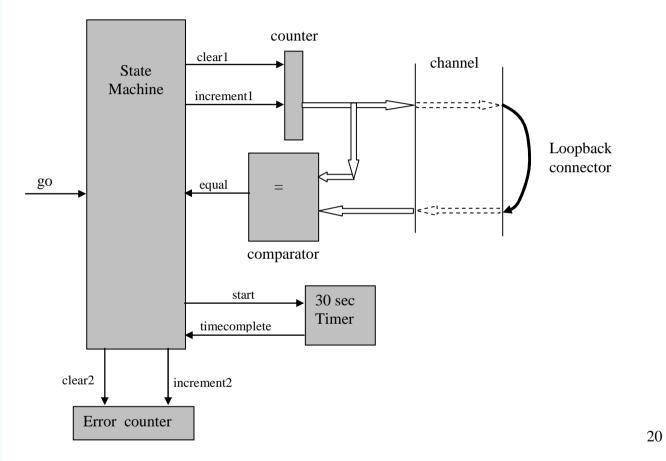




 $Dq = \overline{P}Q + \overline{P}T$

2006 Q2

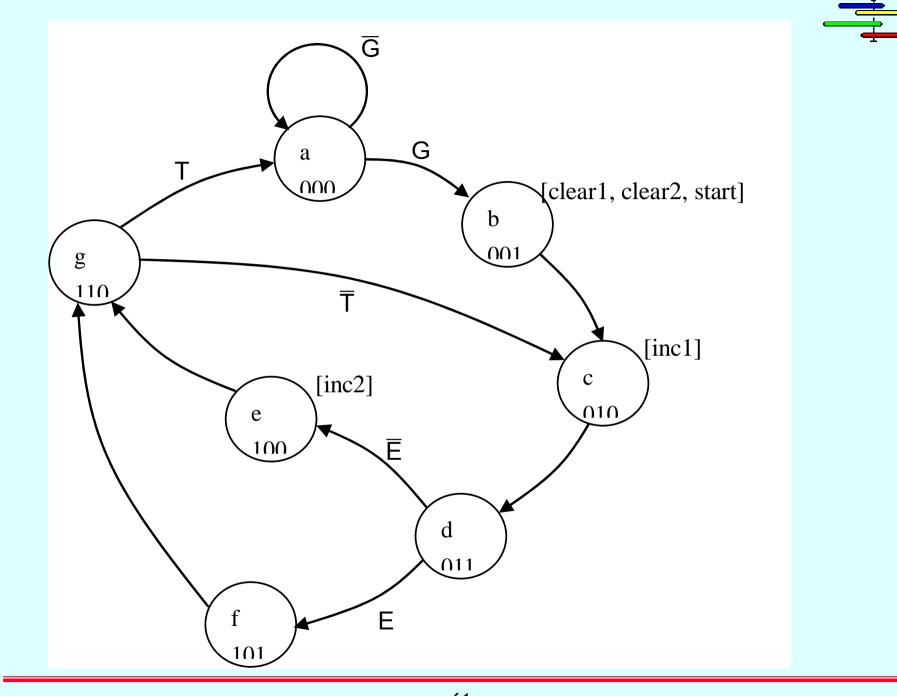
A long communications channel needs to be tested for potential problems. The technique used is to send a sequence of values through the channel and route them back through a loopback connection. The simplest sequence is to send the numbers 0, 1, 2, 3, ...etc. Each sent value is compared to the received value. Any difference will cause an error counter to be incremented. A test session is to be run for 30 seconds. A timer unit will provide this feature. During a test session, a large number of values will be sent through the channel and compared for errors. The block diagram is as follows

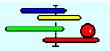


Generate a state transition diagram for this system. Using a Moore machine representation, show the outputs as part of the state machine.

Hence produce a set of excitation equations for a D-type flip-flop implementation.







Transition table

	Target state					
	Р	Q	R	Dp	Dq	Dr
a→a a→b	0 0	0 0	0 1	-	-	- G
b→c	0	1	0	-	1	-
c→d	0	1	1	-	1	1
d→e d→f	1 1	0 0	0 1	Ē E	-	- E
e→g	1	1	0	1	1	-
f→g	1	1	0	1	1	-
g→c g→a	0 0	1 0	0 0	-	⊤ -	-

