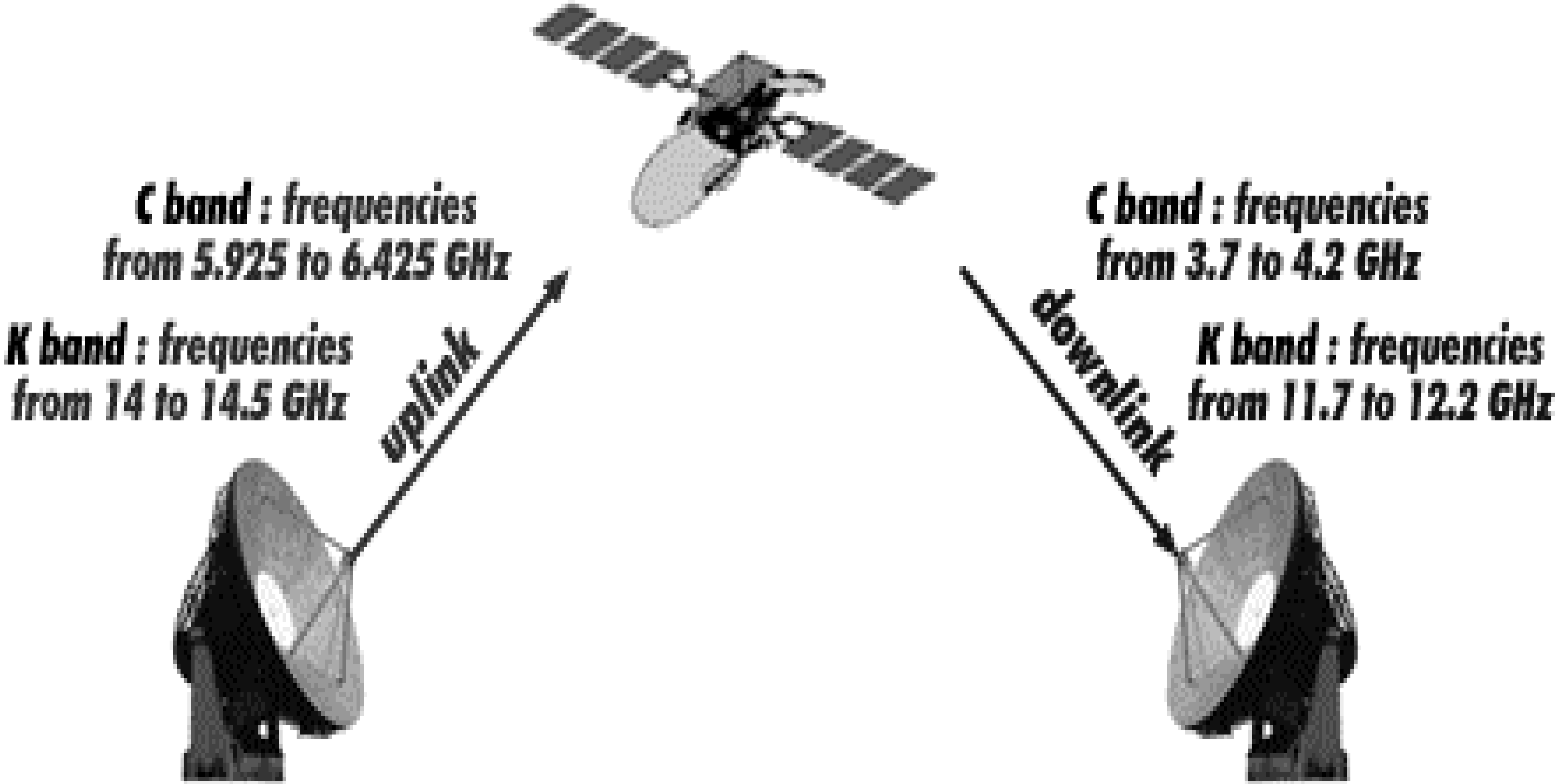
Sinusoidal EM Waves

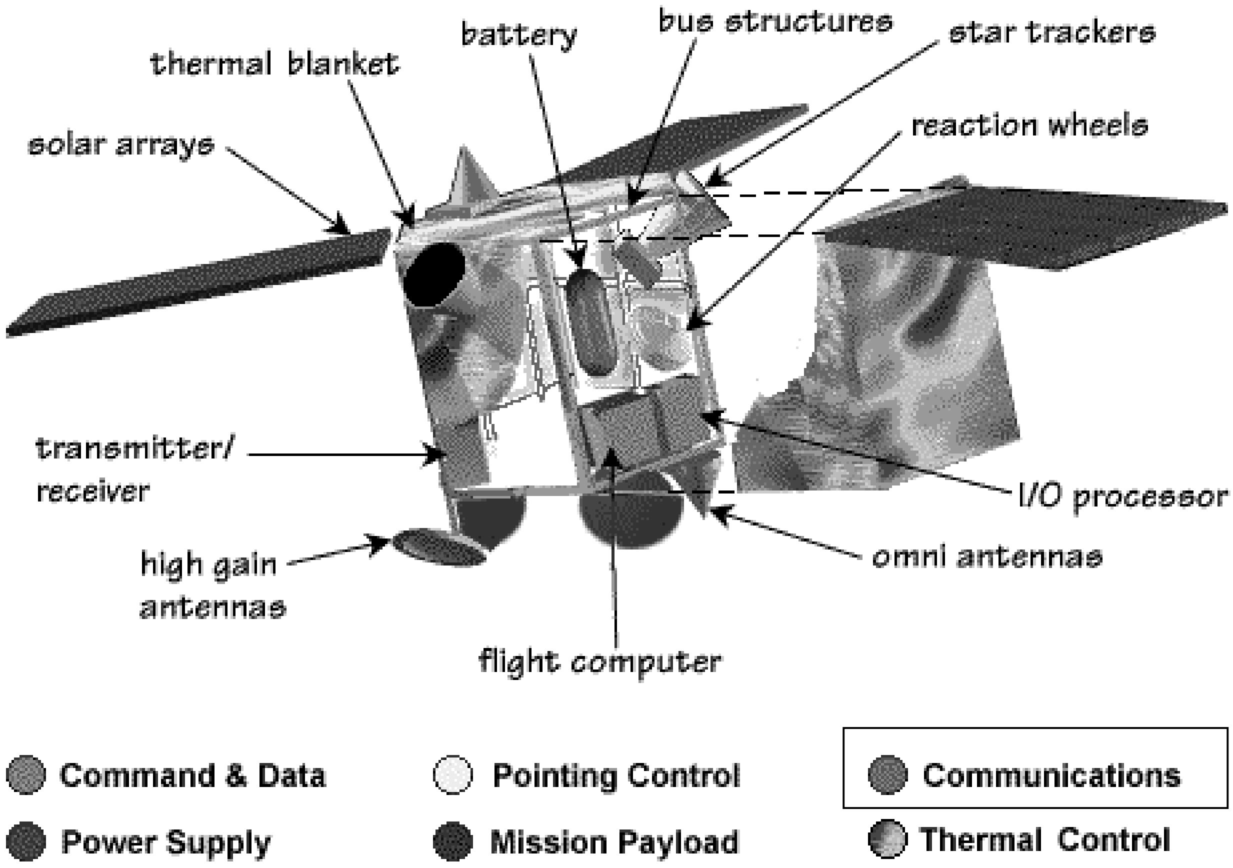
22.2MB1

Dr Yvan Petillot

Introduction



Introduction



Introduction

How to design the communications side of a satellite?

1 Frequency ?

• Optimise propagation ?	Wave equations
• Optimise data rate ?	Signal analysis

2 Antennas?

• Directivity ?	Wave equations
• Gain ?	& Fourier analysis

3 Power budget?

- Linked to 1 & 2
- Poynting vector

Section Contents

- Fourier representation
- Phasors
- Maxwell equations in phasor forms
- Plane wave propagation, phase velocity
- Power flow and Poynting vector
- Propagation in media
  - Propagation in conductive media
  - Propagation in Good dielectric
  - Propagation in Good conductor
  - Skin Depth
  - Power loss in metal

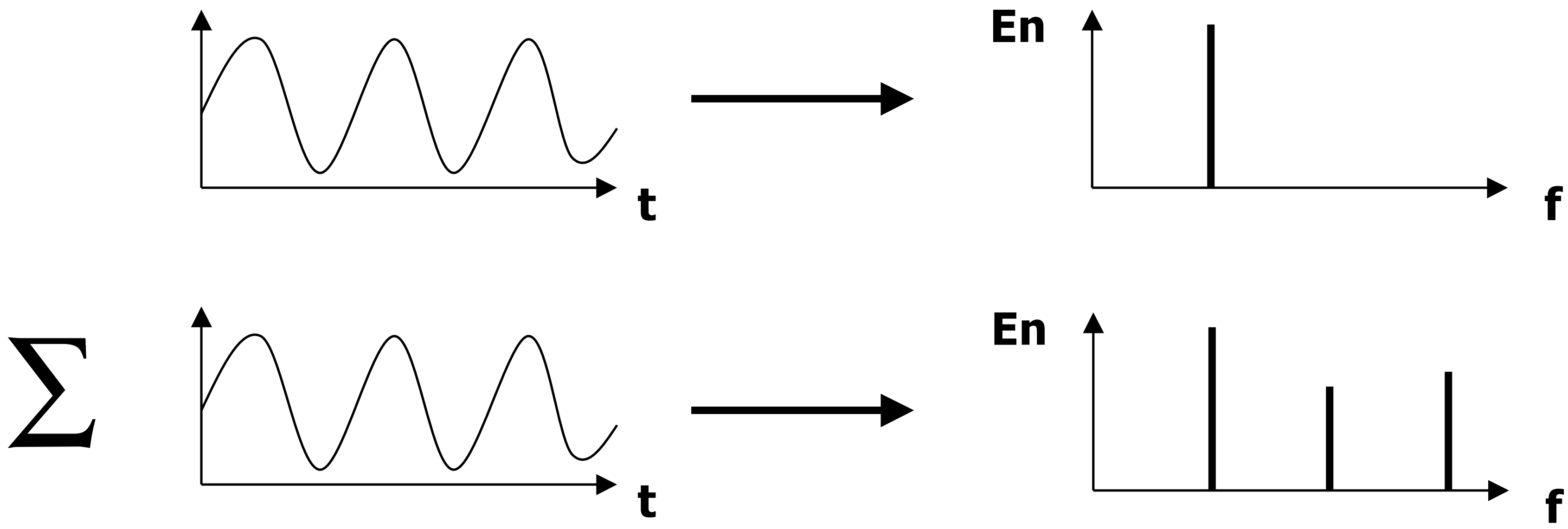
Frequency analysis

Sinusoidal time variation

Time domain	Frequency domain

- Periodic signal:
  - sum of harmonically related sinusoids.

- Energy at frequencies multiple of the fundamental frequency



Fourier series
Synthesis equations

For complex periodic signals

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

Harmonically related complex exponentials
Fourier Analysis

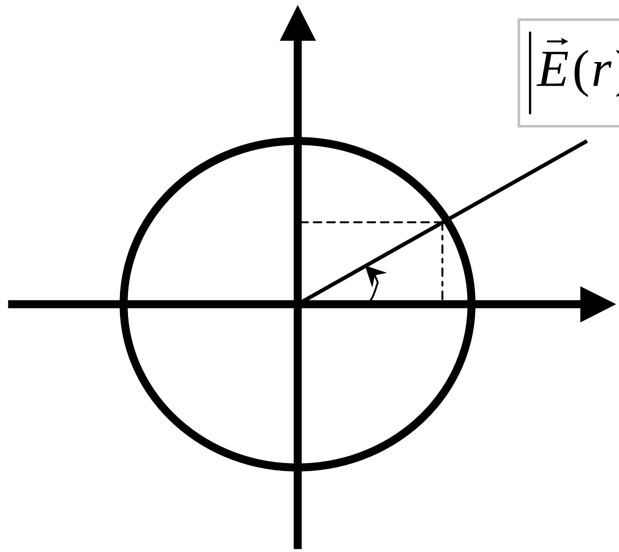
Sinusoidal EM Waves

- We are interested in the behaviour of sinusoidal waves of the form:

$$\vec{E}(r,t) = \mathrm{Re}\left(\vec{E}(r)e^{jwt}\right)$$

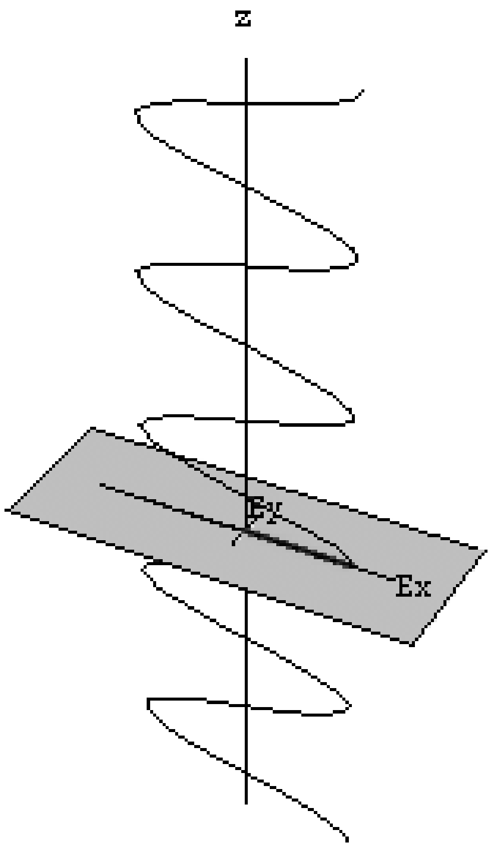
$\vec{E}(r)$ Phasor. Function of space only.

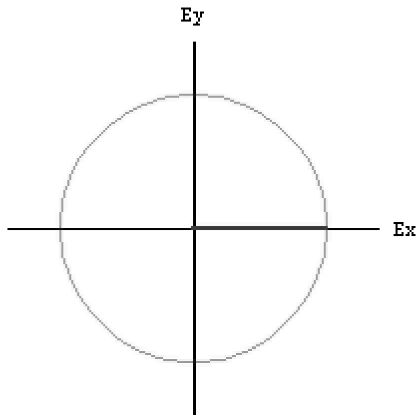
$$\vec{E}(r) = \left|\vec{E}(r)\right|e^{j\Phi}\vec{u}$$



$\left|\vec{E}(r)\right|e^{j\Phi}$

Sinusoidal EM Waves





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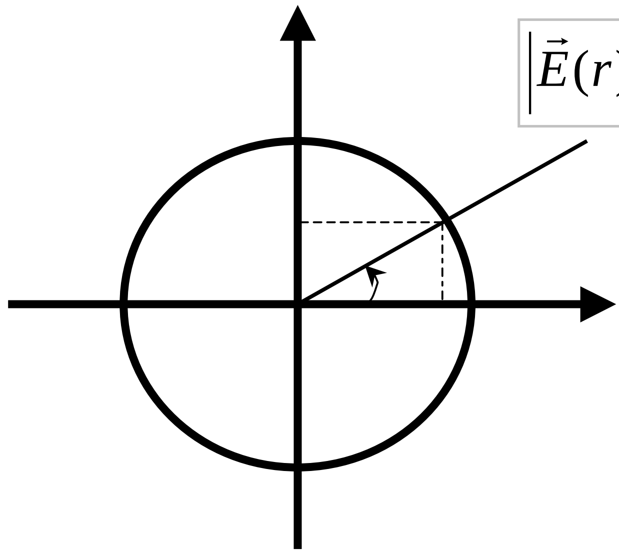
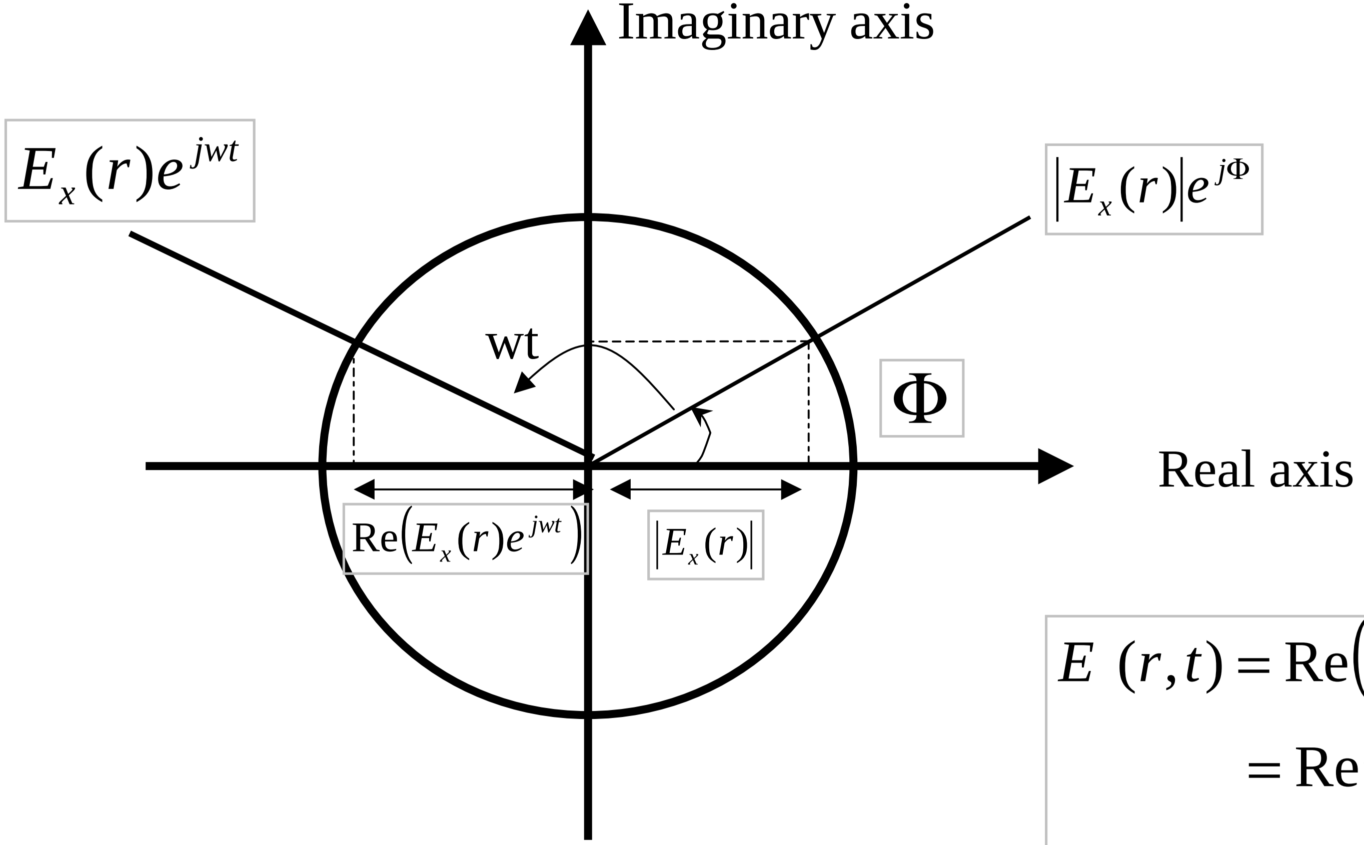


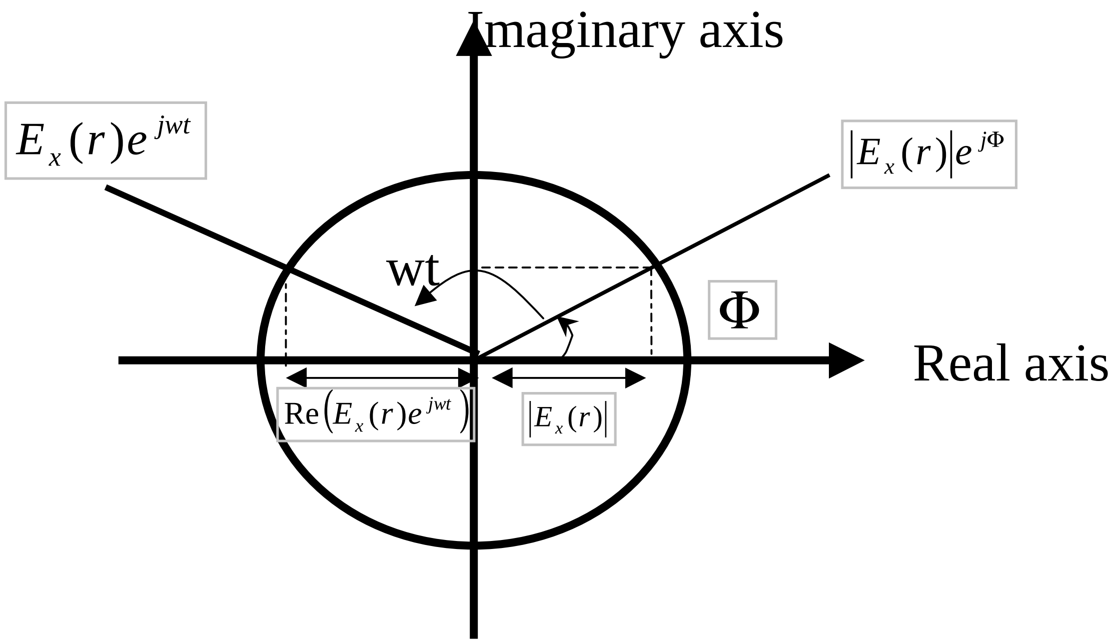
$\left|\vec{E}(r)\right|e^{j\Phi}$

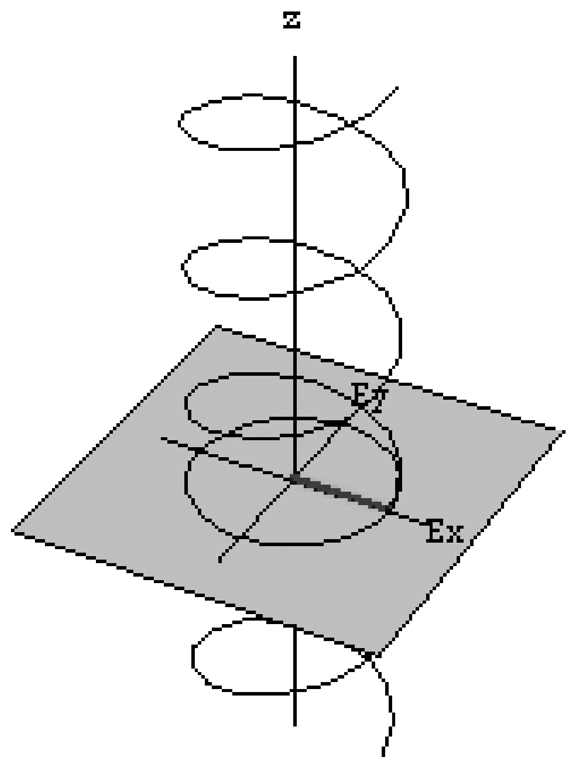
Geometric interpretation



$$
\begin{aligned}
E\ (r,t) &= \mathrm{Re}\left(E_x(r)e^{jwt}\right) \\
&= \mathrm{Re}\left(|E_x(r)|e^{j\Phi}e^{jwt}\right) \\
&= \mathrm{Re}\left(|E_x(r)|e^{j(wt+\Phi)}\right) \\
&= |E_x(r)|\cos(wt+\Phi)
\end{aligned}
$$

Geometric interpretation





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Complex Numbers
Euler's relation

Euler's relation

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

Maxwell's equations in phasor notation

Example

$$\vec{\nabla}\times\vec{H}(r,t)=\vec{J}(r,t)+\frac{\partial\vec{D}(r,t)}{\partial t}$$

Becomes

$$\vec{\nabla}\times\mathrm{Re}\left(\vec{H}(r)e^{jwt}\right)=\mathrm{Re}\left(\vec{J}(r)e^{jwt}\right)+\frac{\partial}{\partial t}\mathrm{Re}\left(\vec{D}(r)e^{jwt}\right)$$

$$\mathrm{Re}\left(\left\{\vec{\nabla}\times\vec{H}(r)-\vec{J}(r)-jw\vec{D}(r)\right\}e^{jwt}\right)=0$$

Finally we have

$$\vec{\nabla}\times\vec{H}=\vec{J}+jw\vec{D}$$

Maxwell's equations in phasor notation

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad\qquad \vec{\nabla}\times\vec{E} = -jw\vec{B}$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \qquad\qquad \vec{\nabla}\times\vec{H} = \vec{J} + jw\vec{D}$$

$$\vec{\nabla}.\vec{D} = \rho \qquad\qquad \vec{\nabla}.\vec{D} = \rho$$

$$\vec{\nabla}.\vec{B} = 0 \qquad\qquad \vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla}.\vec{J} = -\frac{\partial\rho}{\partial t} \qquad\qquad \vec{\nabla}.\vec{J} = -jw\rho$$

$$\vec{D} = \varepsilon\,\vec{E} \qquad \vec{B} = \mu\,\vec{H} \qquad \vec{J} = \sigma\,\vec{E}$$

Plane wave in phasor form

Source free non-conductive medium:

$$\nabla^2 \vec{E}(r,t) - \mu\,\varepsilon \frac{\partial^2 \vec{E}(r,t)}{\partial t^2} = 0$$

$$\vec{E}(r,t) = \vec{E}(r) e^{jwt}$$

$$\frac{\partial}{\partial t}\vec{E}(r,t) = \frac{\partial}{\partial t}\vec{E}(r) e^{jwt} = jw\vec{E}(r) e^{jwt} = jw\vec{E}(r,t)$$

$$\nabla^2 \vec{E} + w^2 \mu\,\varepsilon\,\vec{E} = 0 \qquad \nabla^2 \vec{H} + w^2 \mu\,\varepsilon\,\vec{H} = 0$$

Frequency analysis appearing

Plane wave in phasor form

$$\nabla^2 \vec{E} + w^2 \mu \varepsilon \vec{E} = 0$$

$$\boxed{\nabla^2 \vec{E} + k^2 \vec{E} = 0}$$ with $k^2 = w^2 \mu \varepsilon$

Helmholtz equation

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 \qquad \frac{\partial^2 H_x}{\partial z^2} + k^2 H_x = 0$$

$$\frac{\partial^2 E_y}{\partial z^2} + k^2 E_y = 0 \qquad \frac{\partial^2 H_y}{\partial z^2} + k^2 H_y = 0$$

Plane wave in phasor form

Solution?

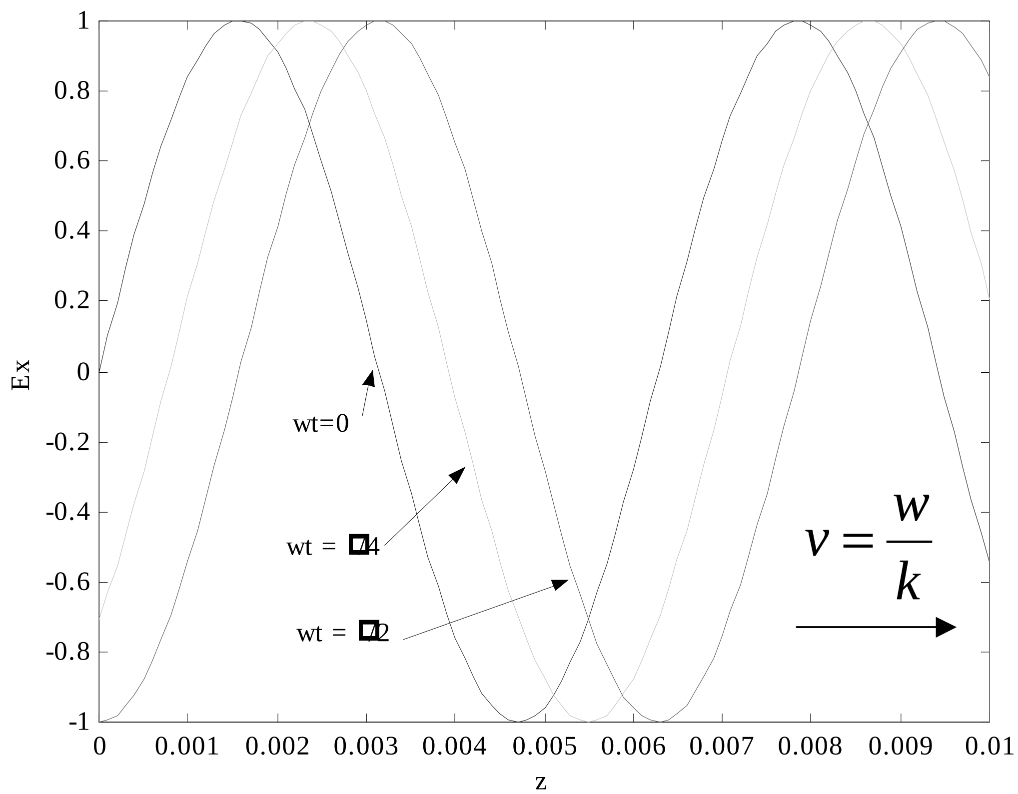
$$E_x = C_1 e^{-jkz} + C_2 e^{jkz}$$

$$\begin{aligned} E_x(z,t) &= \mathrm{Re}\left(E_x(z) e^{jwt}\right) \\ &= \mathrm{Re}\left(C_1 e^{j(wt-kz)} + C_2 e^{j(wt+kz)}\right) \\ &= \mathrm{Re}\left(C_1 e^{jw(t-\frac{k}{w}z)} + C_2 e^{jw(t+\frac{k}{w}z)} \right) \\ &= \underbrace{Af(t-\frac{z}{v})} + \underbrace{Bf(t+\frac{z}{v})} \end{aligned}$$

Wave propagating in z and -z directions

Plane wave in phasor form

$$f(t-\frac{z}{v}) = f(t+\delta t-\frac{z+\delta z}{v}) \quad \Rightarrow \quad \frac{\delta z}{\delta t} = v = \frac{w}{k} \quad \Rightarrow \quad k = \frac{w}{v}$$



$$v = \frac{w}{k}$$

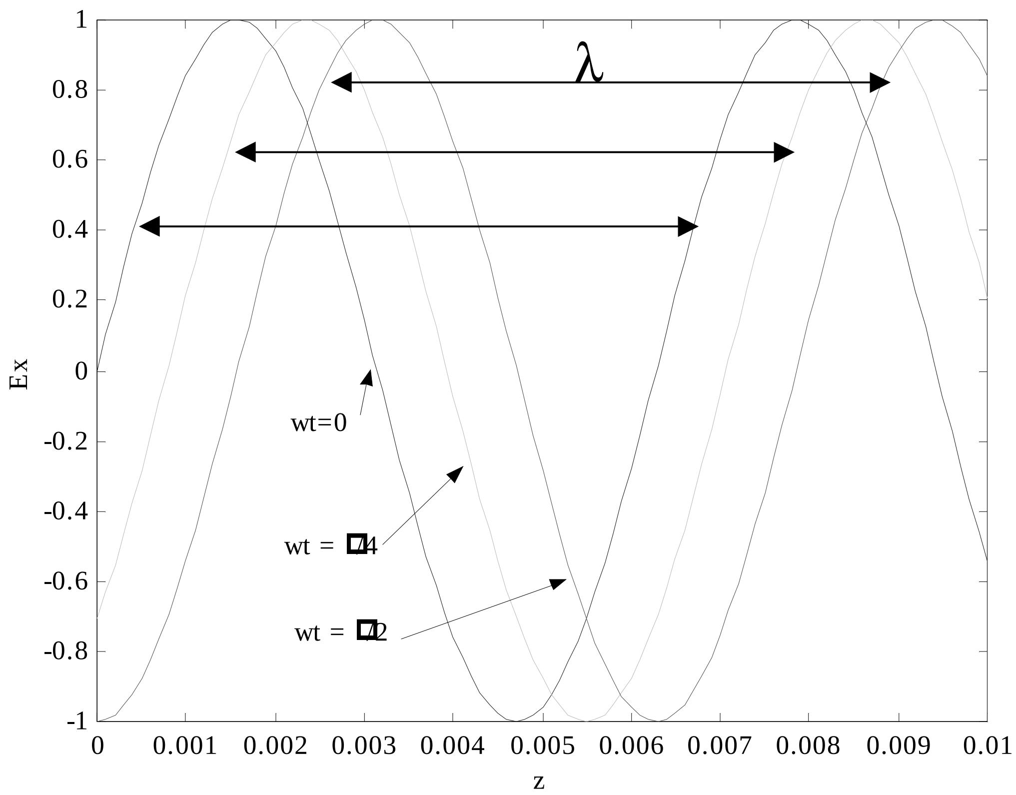
Phase velocity

$$E(z,t) = Ae^{jw(t-\frac{k}{w}z)}$$

Plane wave in phasor form

Wavelength λ:

- independent of time and position in space
- Characteristic of a wave



$$E(z,t) = Ae^{jw(t-\frac{k}{w}z)}$$

$$E(z+\lambda,t) = Ae^{jw(t-\frac{k}{w}(z+\lambda))} = Ae^{jw(t-\frac{k}{w}z)}$$

$$\Downarrow$$

$$k\lambda = 2\pi$$

$$\frac{w}{v}\lambda = 2\pi$$

$$w\lambda = 2\pi \text{ v}$$

$$\downarrow$$

$$w = 2\pi \text{ f} \longrightarrow \text{v} = \text{f}\lambda$$

Plane wave in phasor form

$$k\lambda = 2\pi$$

$$\frac{w}{v}\lambda = 2\pi$$

$$w\lambda = 2\pi \mathrm{v}$$

↓

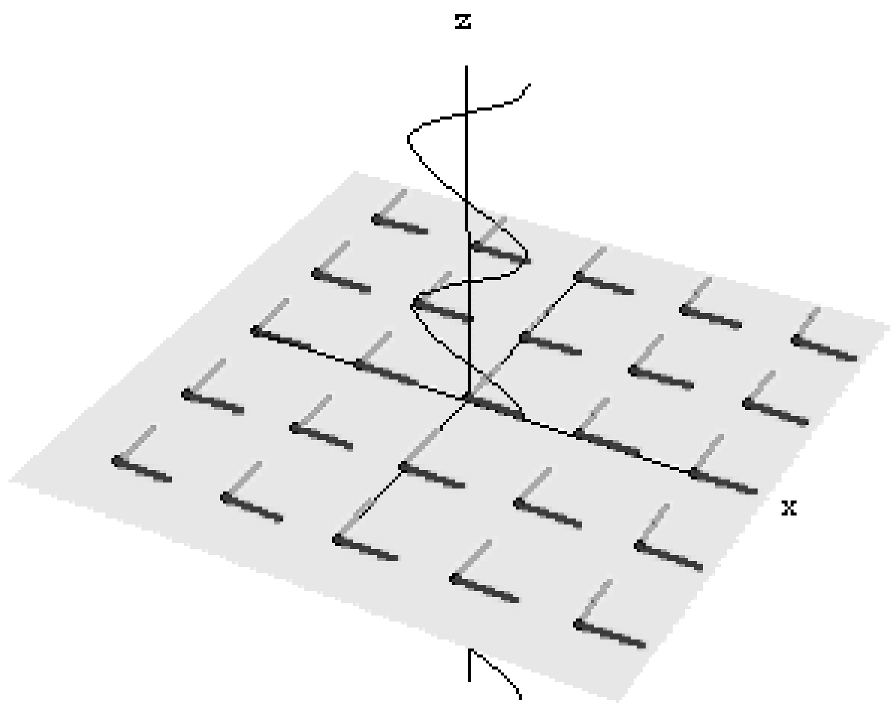
$$w = 2\pi \mathrm{f} \longrightarrow \mathrm{v} = \mathrm{f}\lambda \longleftarrow \begin{cases} \mathrm{k} = \mathrm{w}\sqrt{\mu\,\varepsilon} & \text{Wave number} \\ \lambda = \dfrac{2\pi}{k} & \text{Wavelength} \end{cases}$$

↓

$$\mathrm{v} = \frac{1}{\sqrt{\mu\,\varepsilon}} = \frac{1}{\sqrt{\mu_r\,\varepsilon_r}}\frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = \frac{c}{\sqrt{\mu_r\,\varepsilon_r}}$$

Phase velocity

When do we have plane waves?



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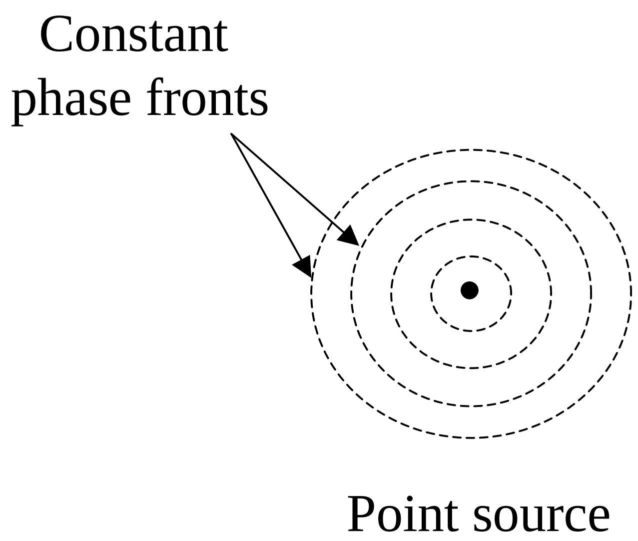
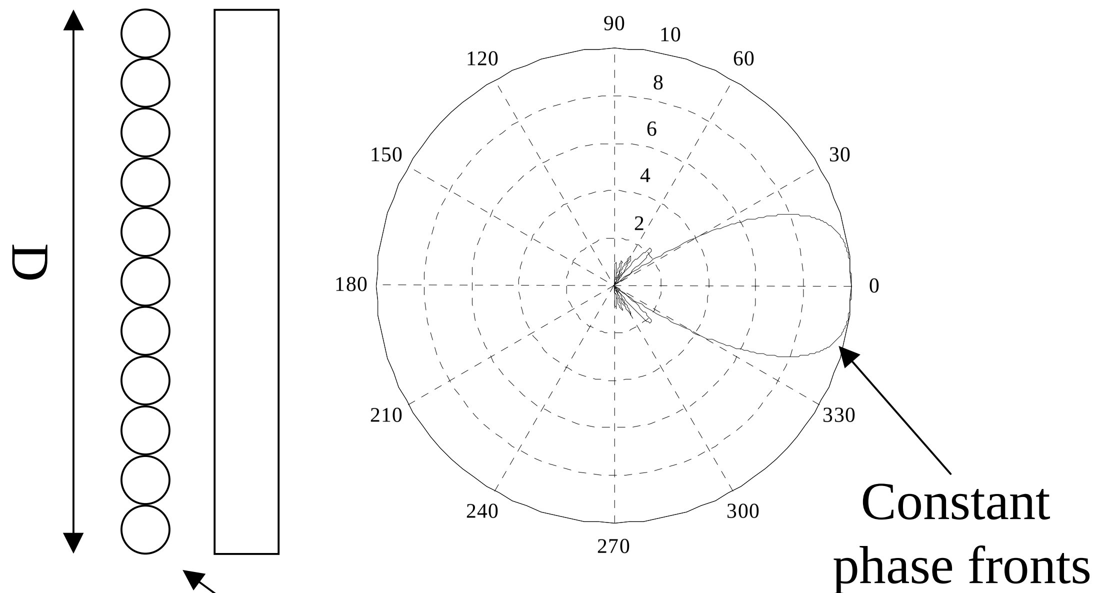
When do we have plane waves?



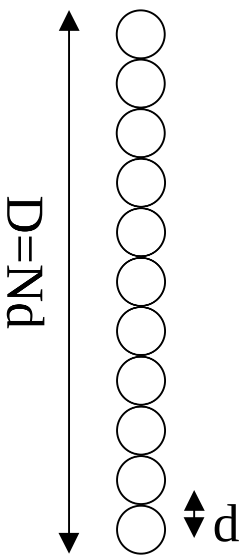


Plane wave when: $r >= \frac{2D^2}{\lambda}$

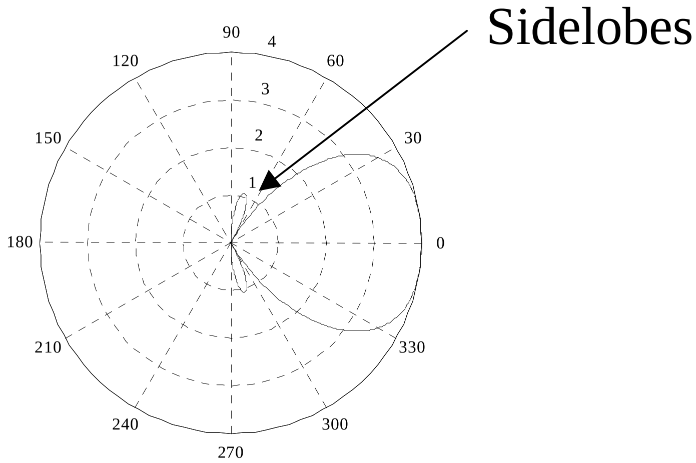
Multiple small receivers (sampling) equivalent to one large antenna

Example: 1 meter antenna and f = 10 GHz
λ = 3 cm and rmin = 66.6 meters

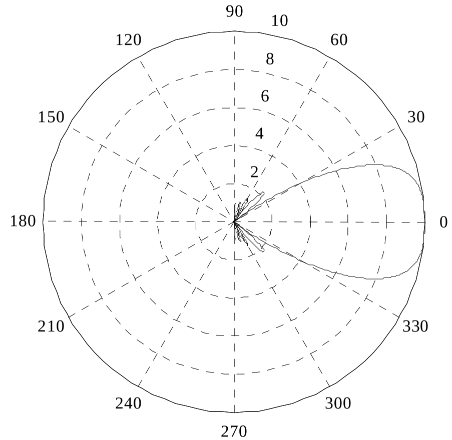
Antennas

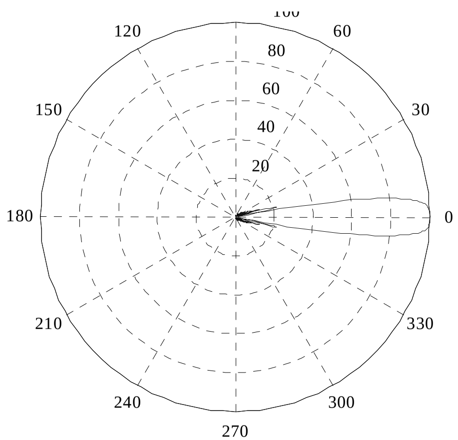


N=4, $d = \lambda/2$

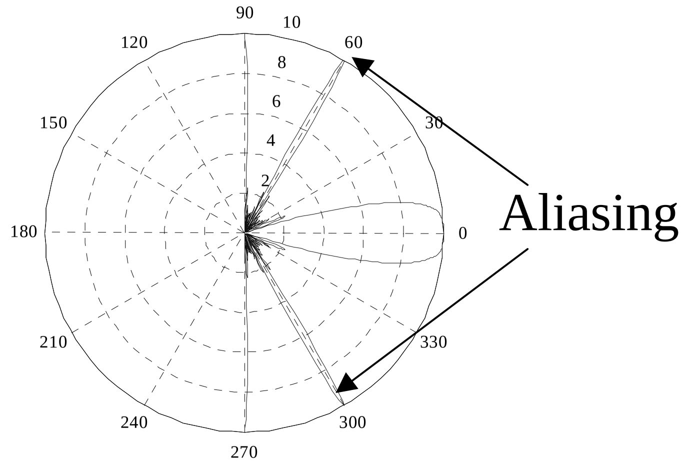


N=10, $d = \lambda/2$





N=100, $d = \lambda/2$



N=100, $d = 2\lambda$

Power Flow

$$\vec{S}(r,t) = \vec{E}(r,t) \times \vec{H}(r,t) \quad \text{Poynting vector}$$

Waves are now sinusoidal and the instantaneous power is less useful

Electric analogy

$$W = \hat{V}\hat{I}$$

$$\hat{V} = \mathrm{Re}(Ve^{jwt}) = \mathrm{Re}(|V|e^{j\theta_v}e^{jwt}) = |V|\cos(wt + \theta_v)$$

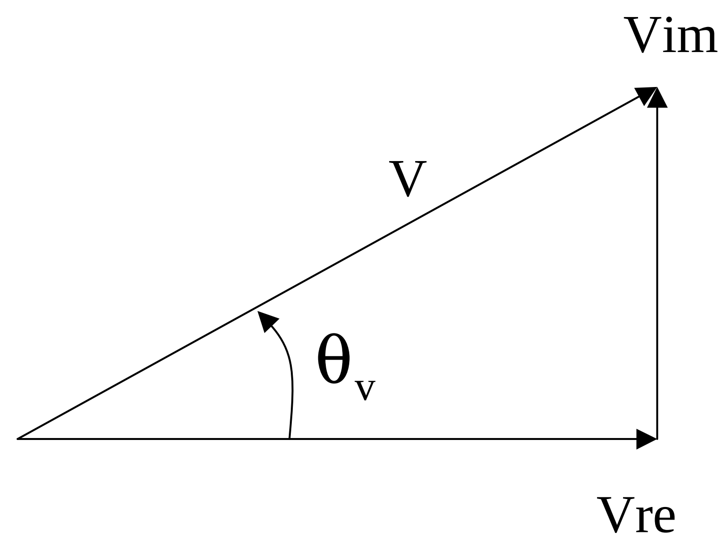
$$\hat{I} = \mathrm{Re}(Ie^{jwt}) = \mathrm{Re}(|I|e^{j\theta_i}e^{jwt}) = |I|\cos(wt + \theta_i)$$

Power Flow

$$W = \hat{V}\hat{I} = |V|\cos(wt+\theta_v)|I|\cos(wt+\theta_i)$$

$$= \frac{|V||I|}{2}\Big[\underbrace{\cos(2wt+\theta_v+\theta_i)}_{\text{Null average over a period}} + \cos(\theta_v-\theta_i)\Big]$$

$$W_{av} = \frac{|V||I|}{2}\cos(\theta_v-\theta_i) = \frac{|V||I|}{2}\cos\theta$$



Power Flow

Electricity

$$W_{react} = \frac{|V||I|}{2}\sin\theta$$

$$W = \frac{1}{2}VI^* = \frac{1}{2}|V||I|e^{j\theta}$$

$$= W_{av} + jW_{react}$$

Em Waves

$$\vec{p} = \frac{1}{2}\vec{E}\times\vec{H}^*$$

E and H are the phasors quantity

$$\vec{p}_{av} = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*)$$

$$\vec{p}_{react} = \frac{1}{2}\mathrm{Im}(\vec{E}\times\vec{H}^*)$$

Power Flow

$$p_z = \frac{1}{2}(E_x H^*_y - E_y H^*_x)$$

But

$$\frac{E_x}{H_y} = -\frac{E_y}{H_x} = \eta = \sqrt{\frac{\mu_0}{\varepsilon}}$$

$$H_x = -\sqrt{\frac{\mu_0}{\varepsilon}} E_y$$

$$H_y = \sqrt{\frac{\mu_0}{\varepsilon}} E_x$$

$$\left\{ \begin{array}{l} H^*_x = -\sqrt{\frac{\mu_0}{\varepsilon}} |E_y| e^{jkz} \\ H^*_y = \sqrt{\frac{\mu_0}{\varepsilon}} |E_x| e^{jkz} \end{array} \right.$$

$$p_z = \frac{1}{2}\left[\sqrt{\frac{\varepsilon}{\mu_0}}|E_x|^2 + \sqrt{\frac{\varepsilon}{\mu_0}}|E_y|^2\right]$$

$$= \frac{1}{2} v\varepsilon |E|^2$$

Power Flow

$$p_z = \frac{1}{2} v \varepsilon |E|^2$$

Electric energy density in the wave? $\frac{1}{2}\frac{\varepsilon |E|^2}{2} = \frac{\varepsilon |E|^2}{4}$ (See notes EM2)

Magnetic energy density in the wave? $\frac{1}{2}\frac{\mu |H|^2}{2} = \frac{\mu |H|^2}{4}$ (See notes EM2)

Total energy density? $\frac{\varepsilon |E|^2}{4} + \frac{\mu |H|^2}{4} = \frac{\varepsilon |E|^2}{4} + \frac{\mu}{4}\left[\sqrt{\frac{\varepsilon}{\mu_0}} E\right]^2 = \frac{1}{2}\varepsilon |E|^2$

The power flow in a plane wave is just the average energy density times the wave velocity

Propagation in conductive media

General wave equation for conducting medium

$$\nabla^2 \vec{E}(r,t) - \mu_0 \, \varepsilon \frac{\partial^2 \vec{E}(r,t)}{\partial t^2} - \sigma \, \mu_0 \frac{\partial \vec{E}(r,t)}{\partial t} = 0$$

$$\nabla^2 \vec{H}(r,t) - \mu_0 \, \varepsilon \frac{\partial^2 \vec{H}(r,t)}{\partial t^2} - \sigma \, \mu_0 \frac{\partial \vec{H}(r,t)}{\partial t} = 0$$

We assume a motion along the z axis

$$\frac{\partial^2 \vec{E}(r,t)}{\partial z^2} - \mu_0 \, \varepsilon \frac{\partial^2 \vec{E}(r,t)}{\partial t^2} - \sigma \, \mu_0 \frac{\partial \vec{E}(r,t)}{\partial t} = 0$$

$$\frac{\partial^2 \vec{H}(r,t)}{\partial z^2} - \mu_0 \, \varepsilon \frac{\partial^2 \vec{H}(r,t)}{\partial t^2} - \sigma \, \mu_0 \frac{\partial \vec{H}(r,t)}{\partial t} = 0$$

Propagation in conductive media

In phasor form it becomes

$$\frac{\partial^2 \vec{E}}{\partial z^2} + \mu_0 \varepsilon w^2 \vec{E} - j\sigma \mu_0 w \vec{E} = 0$$

Helmholtz form

$$\frac{\partial^2 \vec{E}}{\partial z^2} + (\mu_0 \varepsilon w^2 - j\sigma \mu_0 w)\vec{E} = 0 \qquad \frac{\partial^2 \vec{E}}{\partial z^2} - \gamma^2 \vec{E} = 0$$

where $\gamma^2 = jw\mu_0 (\sigma + jw\varepsilon)$

$\gamma = \alpha + j\beta$ With $\alpha > 0$

Solution?

$$\vec{E}(z) = \vec{E}_0 e^{-\gamma z}$$

What if $\alpha < 0$?

Propagation in conductive media

$$\gamma = \alpha + j\beta$$

$$\vec{E}(\mathrm{z}) = \vec{E}_0 e^{-\gamma \mathrm{z}}$$

$$\vec{E}(z,t) = \mathrm{Re}\left(\vec{E}_0 e^{(-\gamma \mathrm{z} + jwt)}\right)$$
$$= \mathrm{e}^{-\alpha \mathrm{z}}\, \mathrm{Re}\left(\vec{E}_0 e^{(jwt - \beta \mathrm{z})}\right)$$

Travelling wave in the z direction attenuated by a factor $e^{-\alpha \mathrm{z}}$

As in the loss-less case, the phase velocity is given by: $v = \dfrac{w}{\beta}$ with $\beta = \dfrac{2\pi}{\lambda}$

Propagation in conductive media

$$\alpha = \mathrm{Re}\left\{ \left[jw\mu_0 (\sigma + jw\varepsilon) \right]^{\frac{1}{2}} \right\}$$

$$\alpha = w\sqrt{\frac{\mu_0\,\varepsilon}{2}\left\{\sqrt{(1+\frac{\sigma^2}{w^2\varepsilon^2})}-1\right\}}$$

Exact formula!

$$\beta = \mathrm{Im}\left\{ \left[jw\mu_0 (\sigma + jw\varepsilon) \right]^{\frac{1}{2}} \right\}$$

$$\beta = w\sqrt{\frac{\mu_0\,\varepsilon}{2}\left\{\sqrt{(1+\frac{\sigma^2}{w^2\varepsilon^2})}+1\right\}}$$

Exact formula!

Good Dielectric case: $\frac{\sigma}{w\varepsilon} << 1$

$\frac{\sigma}{w\varepsilon}$ is called the dissipation factor

Example: Mica at radio frequencies $\frac{\sigma}{w\varepsilon} = 0.0002$

$$\sqrt{(1+\frac{\sigma^2}{w^2\varepsilon^2})} = 1 + \frac{\sigma^2}{2w^2\varepsilon^2} + o(\frac{\sigma^2}{w^2\varepsilon^2}) \quad \text{(Taylor expansion)}$$

$$\alpha = w\sqrt{\frac{\mu_0\,\varepsilon}{2}\left\{(1+\frac{\sigma^2}{2w^2\varepsilon^2})-1\right\}} = w\sqrt{\frac{\mu_0\,\varepsilon}{2}\left\{\frac{\sigma^2}{2w^2\varepsilon^2}\right\}}$$

$$= \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon}} = \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{1}{\varepsilon_r}} = \frac{\sigma}{2}\eta_0\sqrt{\frac{1}{\varepsilon_r}} \quad [\eta_0 = 377\Omega]$$

Good Dielectric case: $\frac{\sigma}{w\varepsilon} << 1$

$$\beta = w\sqrt{\frac{\mu_0\,\varepsilon}{2}\left\{(1+\frac{\sigma^2}{2w^2\varepsilon^2})+1\right\}} = w\sqrt{\mu_0\,\varepsilon\left\{1+\frac{\sigma^2}{4w^2\varepsilon^2}\right\}}$$

$$= \underbrace{w\sqrt{\mu_0\,\varepsilon}}_{\text{Perfect dielectric}}\underbrace{\left(1+\frac{\sigma^2}{8w^2\varepsilon^2}\right)}_{\text{Correction term}}$$

Wave velocity

$$v = \frac{w}{\beta} = \frac{1}{\sqrt{\mu_0\,\varepsilon}\left(1+\frac{\sigma^2}{8w^2\varepsilon^2}\right)}$$

$$= v_0\left(1-\frac{\sigma^2}{8w^2\varepsilon^2}\right)$$

Good Conductor case: $\frac{\sigma}{w\varepsilon} >> 1$

Even at microwave range (~30 GHz) $\frac{\sigma}{w\varepsilon} \approx 3.5\ 10^8$ for Copper

When $\frac{\sigma}{w\varepsilon} >> 1$ we can write

$$\gamma = \sqrt{jw\mu_0\,(\sigma + jw\varepsilon)} = \sqrt{jw\mu_0\,\sigma} = \sqrt{w\mu_0\,\sigma}\angle 45^0$$

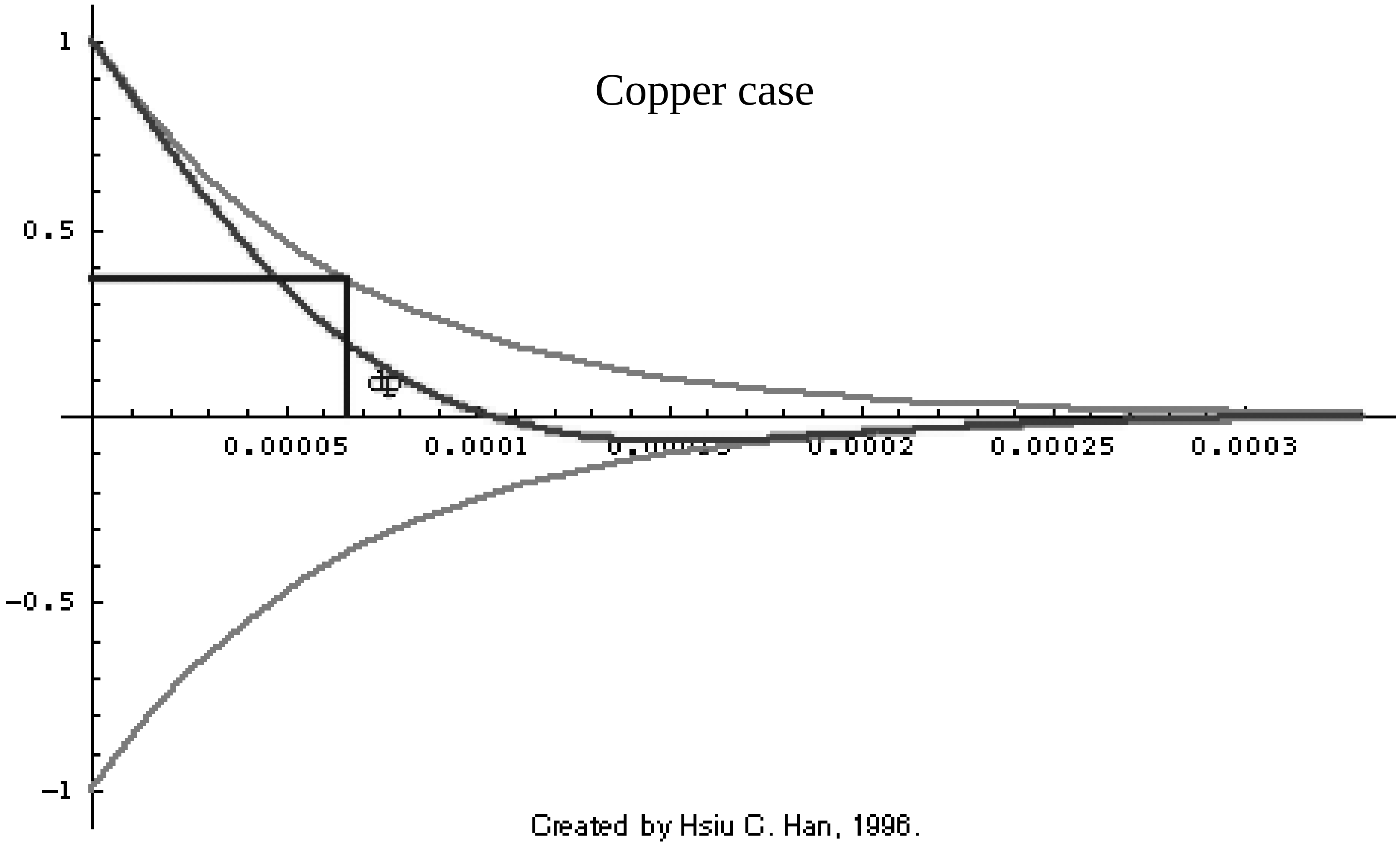
$$\gamma = \alpha + j\beta = \sqrt{\frac{w\mu_0\,\sigma}{2}} + j\sqrt{\frac{w\mu_0\,\sigma}{2}}$$

$$\alpha = \beta = \sqrt{\frac{w\mu_0\,\sigma}{2}}$$

Wave velocity

$$v = \frac{w}{\beta} = \sqrt{\frac{2w}{\mu_0\sigma}}$$

Good Conductor case: $\frac{\sigma}{w\varepsilon} >> 1$



Wave impedance

Wave Impedance

$$\eta = \frac{E}{H} \qquad \nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\frac{\partial E_x}{\partial z} = -jw\mu_0 H_y \qquad -\gamma E_x = -jw\mu_0 H_y$$

$$\frac{E_x}{H_y} = \eta = \frac{jw\mu_0}{\gamma} \qquad \eta = \frac{jw\mu_0}{(\alpha + j\beta)}$$

Good Dielectric case:

$$\eta = \frac{jw\mu_0}{(\alpha + j\beta)} = \frac{jw\mu_0}{\sqrt{jw\mu_0\,(\sigma + jw\varepsilon)}} = \sqrt{\frac{jw\mu_0}{(\sigma + jw\varepsilon)}}$$

Good dielectric: $\frac{\sigma}{w\varepsilon} << 1$

$$\eta = \sqrt{\frac{jw\mu_0}{(\sigma + jw\varepsilon)}} = \sqrt{\frac{\mu_0}{\varepsilon}\frac{1}{(1+\frac{\sigma}{jw\varepsilon})}} = \sqrt{\underbrace{\frac{\mu_0}{\varepsilon}}(1+\underbrace{\frac{j\sigma}{2w\varepsilon}})}$$

Intrinsic impedance

Small reactive component

Good conductor case:

$$\eta = \frac{jw\mu_0}{(\alpha + j\beta)} = \frac{jw\mu_0}{\sqrt{jw\mu_0\,(\sigma + jw\varepsilon\,)}} = \sqrt{\frac{jw\mu_0}{(\sigma + jw\varepsilon\,)}}$$

Good conductor: $\frac{\sigma}{w\varepsilon} >> 1$

$$\eta = \sqrt{\frac{jw\mu_0}{\sigma}} = \sqrt{\frac{w\mu_0}{\sigma}}\angle 45^0$$

Skin Depth

σ not negligable

$$\vec{E}(z,t) = e^{-\alpha z}\,\mathrm{Re}\left(\vec{E}_0 e^{(jwt-\beta z)}\right) \qquad \alpha = w\sqrt{\frac{\mu_0\,\varepsilon}{2}\left\{\sqrt{(1+\frac{\sigma^2}{w^2\varepsilon^2})}-1\right\}}$$

attenuation

Definition: the skin depth δ is defined as the distance in the direction of propagation in which the wave has been attenuated by $\frac{1}{e} \approx 37\%$

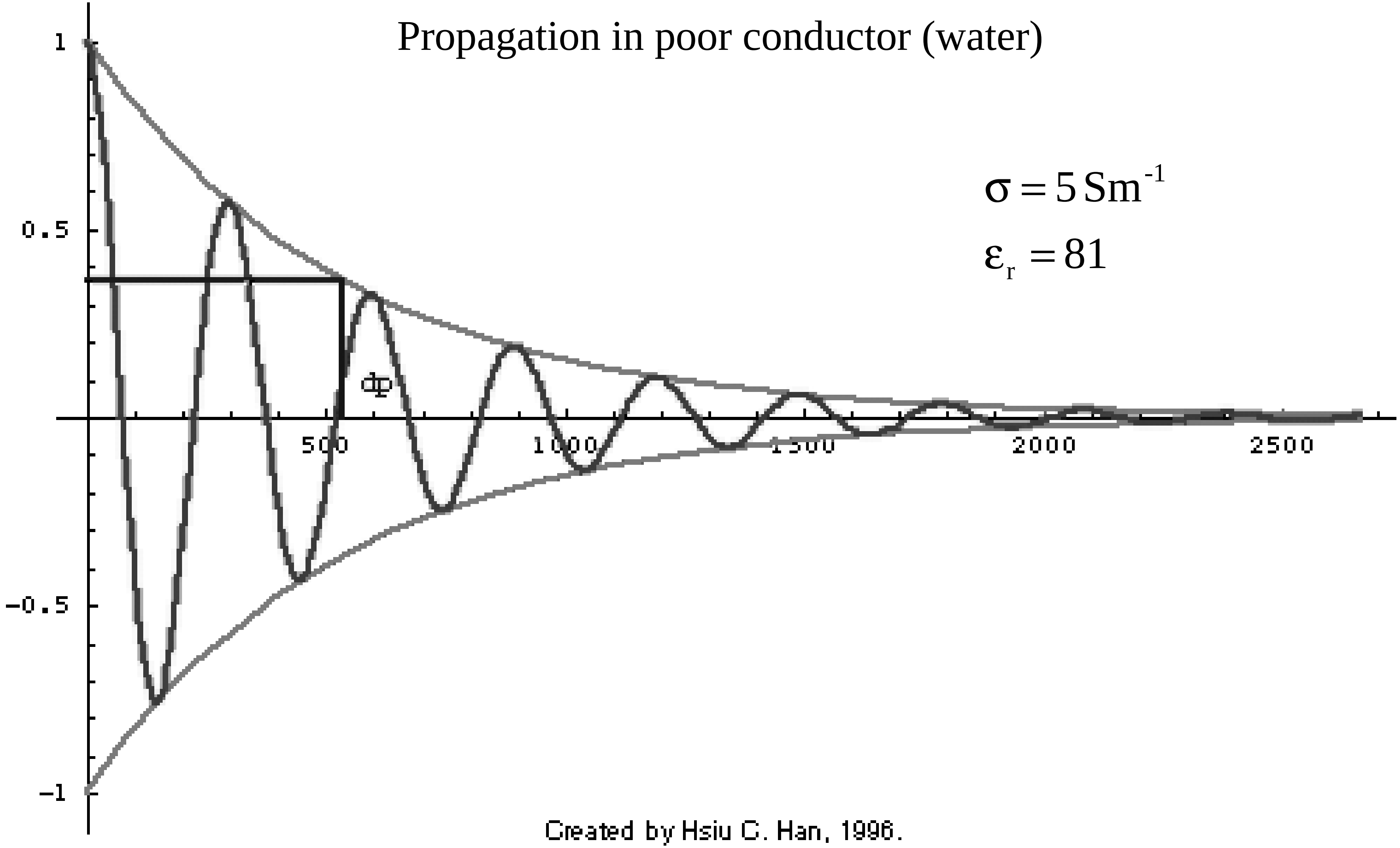
$$\delta = \frac{1}{\alpha} = \frac{1}{w\sqrt{\frac{\mu_0\,\varepsilon}{2}\left\{\sqrt{(1+\frac{\sigma^2}{w^2\varepsilon^2})}-1\right\}}}$$

Good conductor
$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\sigma\mu_0 w}}$

Skin Depth

σ not negligable

Propagation in poor conductor (water)

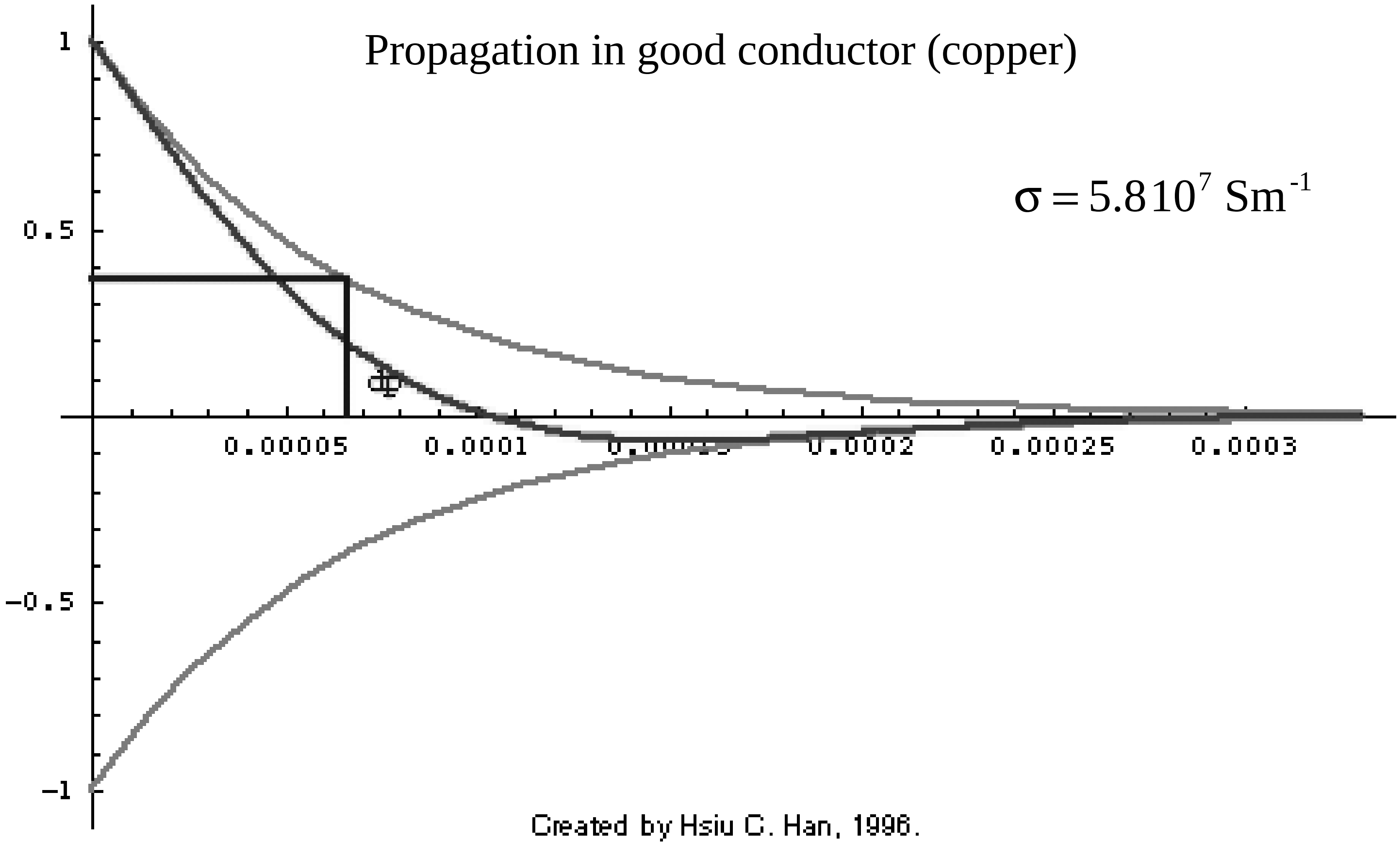


$\sigma = 5\,\mathrm{Sm}^{-1}$

$\varepsilon_r = 81$

Skin Depth

σ not negligable



Skin Depth

σ not negligable

Propagation in good conductor (copper)

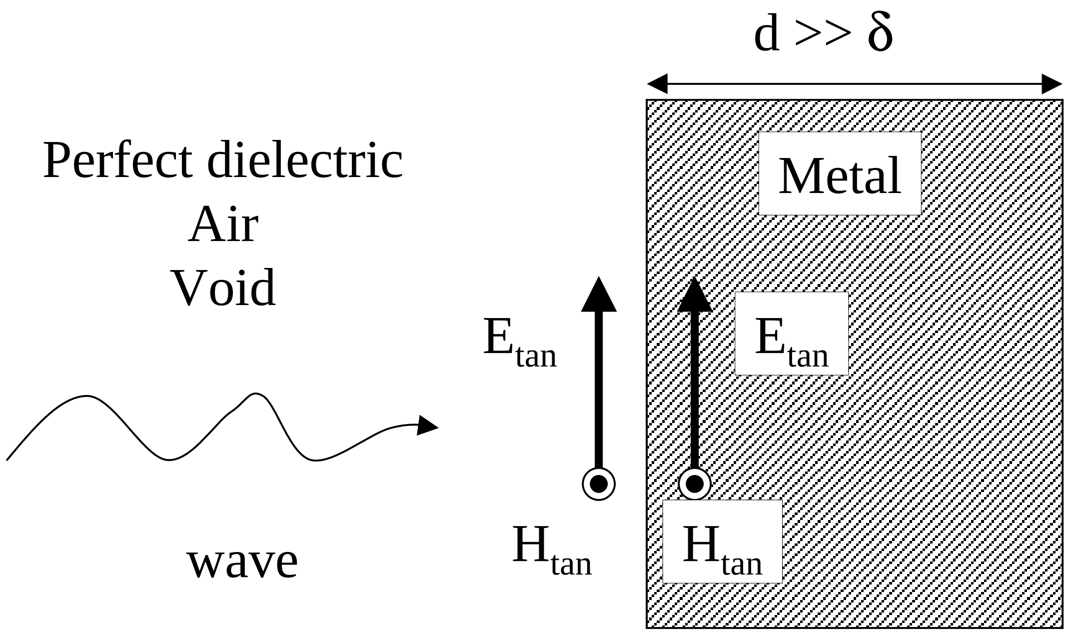
$\sigma = 5.8 10^{7}\ \mathrm{Sm^{-1}}$ $\quad f = 1\mathrm{Mhz}$ $\quad \mu = \mu_0 = 4\pi 10^{-7}\ \mathrm{H}$

Good conductor
$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\sigma \mu_0 w}}$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{5.8 10^{7} 4\pi 10^{-7} 2\pi 10^{6}}} = 0.0667\mathrm{mm}$$

Power Loss in Metal

We have seen (boundary conditions)



$$\frac{E_{tan}}{H_{tan}} = \eta_{metal}$$

$$\eta_{metal} = \sqrt{\frac{jw\mu_0}{(\sigma + jw\varepsilon)}}$$

$$\eta_{metal} = \sqrt{\frac{jw\mu_0}{\sigma}} = \sqrt{\frac{w\mu_0}{\sigma}} \angle 45^0$$

$$p_{av} = \frac{1}{2}\mathrm{Re}(E_{tan} \times H_{tan}^{\ *})$$

Power Loss in Metal

$$\frac{E_{tan}}{H_{tan}} = \eta_{metal} \qquad \eta_{metal} = \sqrt{\frac{jw\mu_0}{\sigma}} = \sqrt{\frac{w\mu_0}{\sigma}}\angle 45^0$$

$$p_{av} = \frac{1}{2}\mathrm{Re}(E_{tan} \times H_{tan}^{*}) = \frac{1}{2}\left|E_{tan}\right|\left|H_{tan}\right|\cos(\pi/4)$$

$$= \frac{1}{2\sqrt{2}}\eta_{metal}\left|H_{tan}\right|^2$$

$$= \frac{1}{2\sqrt{2}}\frac{\left|E_{tan}\right|^2}{\eta_{metal}}$$