Electromagnetics: revision lecture

22.2MB1

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Based on Dr Peter Smith 22.1MA1 material

Section Contents

- Maxwell equations (Integral form)
- Static cases
  - *Electric field, (E-field)*
    - *electric flux, Φ.*
    - *electric flux density, D.*
    - *electric flux intensity, E.*
    - *permittivity, ε.*
  - *Magnetic field, (H-field).*
    - *magnetic flux, Ψ.*
    - *magnetic flux density, B.*
    - *magnetic flux intensity, H.*
    - *permeability, μ.*
    - *Flux linkage, Λ.*
- Back to Maxwell (Dynamic case)

The big picture

Electrostatics
electric field, potential difference, capacitance, charge

Magnetostatics
magnetic field, current, inductance, magnetism

Electromagnetics
induction, emf, radiation

From the beginning….

- The Danish physicist, Hans Christian Oersted, discovered approximately 200 years ago that electricity and magnetism are linked. In his experiment he showed that a current carrying conductor produced a magnetic field that would affect the orientation of a nearby magnetic compass. This principle is used for electromagnets, magnets that can be switched on or off by controlling the current through a wire.
- 3 month later, Faraday had derived a theory…
- Then came Maxwell (19th century)

Maxwell's Equations (Integral Form)

$\oint_C \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$	Maxwell's 1st Equation = Faraday's Law
$\oint_C \vec{H}.d\vec{l} = \int_S \left(J + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$	Maxwell's 2nd Equation = Ampere's Law
$\oint_S \vec{D}.d\vec{S} = \int_v \rho dv$	Maxwell's 3rd Equation = Gauss's Law
$\oint_S \vec{B}.d\vec{S} = 0$	Maxwell's 4th Equation = Conservation of Magnetic Flux

Electrostatics

$\oint_C \vec{E}.d\vec{l} = 0$	Maxwell's 1st Equation = Faraday's Law
$\oint_S \vec{D}.d\vec{S} = \int_v \rho dv$	Maxwell's 3rd Equation = Gauss's Law No Electric fields without charges

Magnetostatics

$\oint_C \vec{H}.d\vec{l} = \int_S \vec{J}.d\vec{S}$	Maxwell's 2nd Equation = Ampere's Law No magnetic Field without currents
$\oint_S \vec{B}.d\vec{S} = 0$	Maxwell's 4th Equation = Conservation of Magnetic Flux

Electric Variables and Units

Field intensity, E (V/m)
Flux density, D (C/m^2)
Flux, Ψ (Coulombs) (C)
Charge, Q (C)
Line charge density, λ (C/m)
Surface charge density, σ (C/m^2)
Volume charge density, ρ (C/m^3)
Capacitance, C (Farads) (F)
C = Q / V (F)

Electric field intensity

- The force experience by any two charged bodies is given by Coulomb’s law.
- Coulomb’s force is inversely proportional to the square of distance.

$$\boxed{\vec{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\vec{a}_r \quad (Newtons)}$$

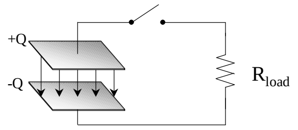
$$\boxed{\vec{E} = \frac{\vec{F}}{q} \quad (N/C) \text{ or } (V/m)}$$

- Electric field intensity, E, is defined in terms of the force experienced by a test charge located within the field.
- We could solve all of the electrostatics with this law as:

$$\boxed{\vec{E}(m) = \frac{1}{4\pi\varepsilon_0}\iiint_{\substack{all\\space}} \frac{\rho(m')}{r_{m,m'}^2}\vec{a}_{m,m'}dV}$$

Energy in an electrostatic system.

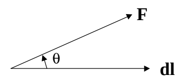
- Like charges are repelled due to Coulomb’s force.
- Opposing charges are attracted due to Coulomb’s force.
- The plates of a capacitor are oppositely charged.
- A mechanical force maintains the charge separation.
- In this way a charged capacitor stores **potential energy**.
- The potential energy is released when the switch is closed and the capacitor releases the charge, as current, to a load.



Work of a force

- Work (or energy) is a function of the path travelled by an object and the force vector acting upon the object.
- Work (or energy), W, is defined as the line integral of force.

$$\boxed{W = \int_l \vec{F}.\vec{dl} \quad (Joules)}$$



- If the path is straight and the force vector is constant then the equation simplifies to:-

$$\vec{F}.\vec{l} = Fl\cos\theta \quad (J)$$

Potential difference law

- Potential energy per unit charge, W/Q, is used in electrical systems.
- Potential energy per unit charge is defined as the line integral of electric field intensity, E.
- **Potential energy per unit charge is known as potential difference.**
- Potential difference is a scalar.

$$\boxed{V = -\int_l \vec{E}.\vec{dl} \quad (Joules/Coulomb) \quad (Volts)}$$

- V = potential difference (V)
- E = field intensity (V/m)

$$\vec{E} = \frac{\vec{F}}{Q}$$

Gauss’s Law
(Incorporating volume charge density)

$\oint_C \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$	$\oint_S \vec{D}.d\vec{S} = \int_v \rho dv$
$\oint_C \vec{H}.d\vec{l} = \int_S \left(J + \frac{\partial \vec{D}}{\partial t}\right)d\vec{S}$	$\oint_S \vec{B}.d\vec{S} = 0$

- Maxwell’s 3rd equation is also known as Gauss law.
- Electric flux begins on bodies of positive charge.
- Electric flux ends on bodies of negative charge.
- Charge is separable and can be enclosed by a closed surface.

$$\boxed{\Psi = \oint_S \vec{D}.d\vec{S} = \int_v \rho dv \quad (Coulombs)}$$

- Ψ = electric flux (C)
- D = electric flux density (C/m^2)
- ρ = volume charge density (C/m^3)

Gauss's Law
(Basic electrostatic form)

- **Electric flux is equal to the charge enclosed by a closed surface.**
- The closed surface is known as a Gaussian surface
- Integrate the flux density over the Gaussian surface to calculate the flux.
- The flux does not depend on the surface! Use the right one!

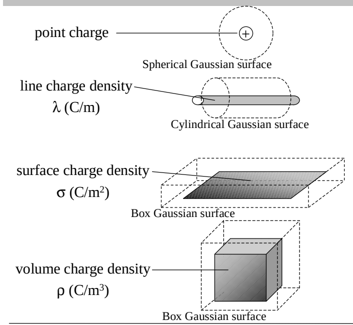
$$\boxed{\Psi = \oint_S \vec{D}.d\vec{S} = Q_{enc}}$$

Gaussian surface

- The **enclosed charge** can be either

a point charge
or
a charge density such as, λ, σ or ρ.

Charge and charge densities



$$Q_{enc} = Q$$

$$Q_{enc} = \int_l \lambda\, dl$$

$$Q_{enc} = \int_s \sigma\, ds$$

$$Q_{enc} = \int_v \rho\, dv$$

Gauss's Law
(General form)

- If the volume of interest has a combination of the above four types of charge distribution then it is written as:-

$$\boxed{\oint_S \vec{D}.d\vec{S} = \underbrace{\sum_1^N Q_N}_{\text{point charges}} + \underbrace{\int_l \lambda\, dl}_{\text{line charges}} + \underbrace{\int_s \sigma\, ds}_{\text{surface charges}} + \underbrace{\int_v \rho\, dv}_{\text{volume charges}}}$$

Electric Field Intensity

- Electric flux density is equal to the product of the permittivity and the electric field intensity, E.

$$\boxed{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} \quad (C/m^2)}$$

- D = Electric Flux Density (C/m²)
- ε_0 = Permittivity of free space = $8.854\times(10)^{-12}$ (F/m)
- ε_r = Relative permittivity of the dielectric material
- E = Electric Field Intensity (V/m)

- D is independent of the material. E isn't

Capacitance

- Capacitance is the ratio of charge to electric potential difference.

$$\boxed{C = \frac{Q}{V} \quad (F)}$$

- C = capacitance (Farads)
- Q = capacitor charge (Coulombs)
- V = potential difference (Volts)

- Energy stored in a condensator:

$$\boxed{W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} \quad (J)}$$

Electrostatics Memento

Force

$$\boxed{\vec{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\vec{a}_r \quad \text{(Newtons)}}$$

$$\boxed{\vec{F} = q\,\vec{E}}$$

Field and Flux

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{a}_r \quad \text{(Newtons)}}$$

$$\boxed{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} \quad (C/m^2)}$$

Work

$$\boxed{W_{A\to B} = \int_A^B \vec{F}.\overrightarrow{dl} = q(V_A - V_B) \quad \text{(Joules)}}$$

Potential

$$\boxed{V = -\int_l \vec{E}.d\vec{l} \quad \text{(Volts)}}$$

$$\boxed{\vec{E} = -\overrightarrow{\text{grad}}\, V}$$

Gauss theorem

$$\boxed{\Psi = \oint_S \vec{D}.d\vec{S} = \int_v \rho dv \quad (Coulombs)}$$

Electrostatics

$$\oint_C \vec{E}.d\vec{l} = 0$$ Maxwell's 1st Equation = Faraday's Law

$$\oint_S \vec{D}.d\vec{S} = \int_v \rho\, dv$$ Maxwell's 3rd Equation = Gauss's Law
No Electric fields without charges

Magnetostatics

$$\oint_C \vec{H}.d\vec{l} = \int_S \vec{J}.d\vec{S}$$ Maxwell's 2nd Equation = Ampere's Law
No magnetic Field without currents

$$\oint_S \vec{B}.d\vec{S} = 0$$ Maxwell's 4th Equation = Conservation of Magnetic Flux

Magnetic Variables and Units

Field intensity, H (A/m)
Flux density, B (Wb/m^2) (Tesla)
Flux, Φ (Webers) (Wb)
Current, I (A)
Current density, J (A/ m^2)
Inductance, L (Henries) (H)
L = Λ / I (Weber-turns/Ampere)
Flux linkage, Λ (Wb-turns)

Magnetic Force (Lorentz Force)

$$\vec{F} = q(\vec{v}\times\vec{B}) \quad \text{(Newtons)}$$

$$\vec{F} = q(\int i\, \overrightarrow{dl}\times\vec{B})$$

Biot Savart Law

$$\vec{B} = \int \frac{\mu_0}{4\pi r^2} I\, \overrightarrow{dl}\times\vec{a}_r$$

Could be used to find any magnetic field
NO MAGNETIC FIELD WITHOUT CURRENT

Current and current density

- The current passes through an area known as the **spanning surface**.
- The current is calculated by integrating the current density over the spanning surface.

$$\oint_S \vec{J}.\overrightarrow{dS} = \text{total current} = I_T$$

Spanning surface

- I_T is also known as the **enclosed current**, I_{enc}.
- Enclosed current signifies the current enclosed by the spanning surface.

Ampere's Law (Basic magnetostatic form)

- The field around a current carrying conductor is equal to the total current enclosed by the closed path integral.
- This equation is the starting point for most magnetostatic problems.

$$\oint_l \vec{H}.d\vec{l} = I_{enc}$$

Ampere's Law (General magnetostatic form)

$$\oint_C \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S} \qquad \oint_S \vec{D}.d\vec{S} = \int_v \rho\, dv$$

$$\oint_C \vec{H}.d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{S} \qquad \oint_S \vec{B}.d\vec{S} = 0$$

- Maxwell's 2nd equation is also known as Ampere's law.
- In magnetostatics the field is not time dependant.
- Therefore there is no displacement current:-

$$J_{disp} = \frac{\partial D}{\partial t} = 0\ (A/m^2)$$

- Therefore

$$\oint_l \vec{H}.d\vec{l} = \int_S \vec{J}.d\vec{S}$$

- H = magnetic field intensity (A/m)
- J = conduction current density (A/m^2)

Flux density

- Magnetic flux density is equal to the product of the permeability and the magnetic field intensity, H.
- Magnetic flux density can simplify to flux divided by area.

$$\boxed{\vec{B} = \mu_0 \mu_r \vec{H} \quad (Wb/m^2) \quad (Tesla)}$$

$$\boxed{\vec{B} = \frac{\Phi}{A} \quad (T)}$$

Equation opposite assumes flux density is uniform across the area and aligned with the unit normal vector of the surface!

- B = Magnetic flux density (T)
- μ_0 = Permeability of free space = $4\pi \times (10)^{-7}$ (H/m)
- μ_r = Relative permeability of the magnetic material
- H = Magnetic field intensity (A/m)
- Φ = Magnetic flux (Wb)

Flux law

$$\oint_C \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S} \qquad \oint_S \vec{D}.d\vec{S} = \int_v \rho\, dv$$

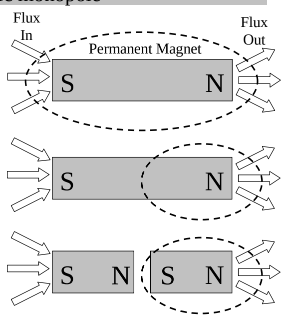
$$\oint_C \vec{H}.d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{S} \qquad \oint_S \vec{B}.d\vec{S} = 0$$

- Maxwell's 4th equation is the magnetic flux law.
- Unlike electric flux, magnetic flux does not begin at a source or end at a sink. The integral equation reflects that **magnetic monopoles do not exist**.

$$\boxed{\Phi = \oint_S \vec{B}.\overrightarrow{dS} = 0 \quad (Webers)}$$

Non-existence of a magnetic monopole

- Try to use the flux law to measure the flux from the north pole of a magnet,
- **Gaussian surface encloses the whole magnet and there is no net flux**.



- Split the magnet in half to produce a net flux?

- Each halved magnet still has a north and a south pole.
- Still no net flux.

Summary of the flux law

$$\boxed{\Phi = \oint_S \vec{B}.\overrightarrow{dS} = 0}$$

- The flux law expresses the fact that magnetic poles cannot be isolated.
- Flux law states that the total magnetic flux passing through a Gaussian surface (*closed surface)* is equal to zero.
- Magnetic flux, Φ, has no source or sink, it is continuous.
- Note, electric flux, Ψ, begins on +ve charge and ends on -ve charge.
- Electric flux equals the charge enclosed by a Gaussian surface. Charge can be isolated.

Calculating flux

$$\boxed{\Phi = \int_S \vec{B}.\overrightarrow{dS}}$$

- If the surface is not closed then it is possible to calculate the flux passing through that surface.
- An example is the flux passing normal through the cross section of an iron core.

$$\Phi = BA$$

Inductance

- The inductance is the ratio of the flux linkage to the current producing the flux and is given as:-

$$L = \frac{\Lambda}{I} \quad (Henries)$$

- Note: for an N-turn inductor:-

$$L = \frac{N\Phi}{I} = \frac{\mu N^2 A}{l} \quad (H)$$

Faraday's Law

$\oint_C \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$	$\oint_S \vec{D}.d\vec{S} = \int_v \rho\, dv$
$\oint_C \vec{H}.d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{S}$	$\oint_S \vec{B}.d\vec{S} = 0$

- Maxwell's 1st equation is also known as Faraday's law
- The term on the right-hand side represent rate of change of magnetic flux.

$$\frac{d}{dt}\int_S \vec{B}.d\vec{S} = \frac{d}{dt}\Phi$$

- The term on the left represents the potential difference between two points A and B.

$$V_{AB} = -\int_B^A \vec{E}.\overrightarrow{dl}$$

- This gives Faraday's law as:-

$$V_{AB} = -\frac{d\Phi}{dt}$$

Faraday's Law and e.m.f

- Three ways to induce a voltage in a circuit :-

1. Vary the magnetic flux with respect to time.
- Use an A.C. current to magnetise the magnetic circuit.
- Use a moving permanent magnet.

2. Vary the location of the circuit with respect to the magnetic flux.
- Move the coil with respect to the magnetic field.

3. A combination of the above.

Lenz's Law

- Lenz's Law
  - *"The emf induced in a circuit by a time changing magnetic flux linkage will be of a polarity that tends to set up a current which will oppose the change of flux linkage."*
- The notion of Lenz's law is a particular example of the Conservation of Energy Law, whereby every action has an equal and opposite reaction.
- **Analogous to inertia in a mechanical system.**
- Consider if Lenz's law did not exist.
  - Now the induced emf sets up a current which aids the change of flux linkage. This would mean that the induced emf in the secondary coil would increase ad infinitum because it would be continually reinforcing itself. (Contravenes the conservation of energy law!)

Duality

- A duality can be recognised between magnetic and electric field theory.
- Electrostatics → E-field due to stationary charge.
- Magnetostatics → H-field due to moving charge.

Electric	Magnetic
Field intensity, E (V/m)	Field intensity, H (A/m)
Flux density, D (C/m²)	Flux density, B (Wb/m²) (Tesla)
Flux, Ψ (C)	Flux, Φ (Webers) (Wb)
Charge, Q (C)	Current, I (A)
Capacitance, C (Farad) (F)	Inductance, L (Henries) (H)
C = Q/V (Coulombs/Volt)	L = Λ / I (Weber-turns/Ampere)
	Flux linkage, Λ (Wb-turns)

What causes what...

- A current carrying conductor will produce a magnetic field around itself.
- Bodies of electric charge produce electric fields between them.
- A time-varying electric current will produce both magnetic and electric fields, this is better known as an electromagnetic field.

direct current carrying conductor	magnetic field
system of charges	electric field
alternating current carrying conductor	electromagnetic field. **E and B are linked**

Maxwell's Equations

- The majority of this course is based around what is known as Maxwell's equations. These equations summarise the whole electromagnetic topic. **James Clerk Maxwell** (1831-1879), Scottish physicist, who unified the four fundamental laws discovered experimentally by his predecessors by adding the abstract notion of **displacement current** that enables theoretically the idea of wave propagation, (see *Treatise on Electricity & Magnetism*).
- Prior to Maxwell a number of experimentalists had been developing their own laws, namely:
  - *Andre Marie Ampere (1775-1836)*
  - *Michael Faraday (1791-1867)*
  - *Karl Friedrich Gauss (1777-1855)*

Maxwell's Equations (Differential Form)

$\oint_C \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$	$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\oint_C \vec{H}.d\vec{l} = \int_S \left(J + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$	$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
$\oint_S \vec{D}.d\vec{S} = \int_v \rho dv$	$\vec{\nabla}.\vec{D} = \rho$
$\oint_S \vec{B}.d\vec{S} = 0$	$\vec{\nabla}.\vec{B} = 0$

Relation between J and ρ

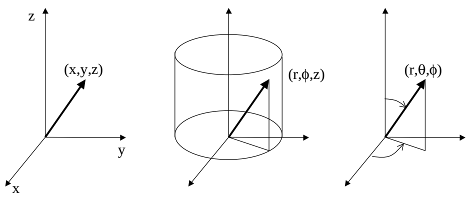
$$\iint \vec{J}.\overrightarrow{dS} = \frac{\partial}{\partial t}\iiint \rho dV$$

Conservation of the charge

$$\vec{\nabla}.\vec{J} = \frac{\partial \rho}{\partial t}$$

Co-ordinate Systems

- Using the appropriate coordinate system can simplify the solution to a problem. In selecting the correct one you should be looking to find the natural symmetry of the problem itself.



CARTESIAN CYLINDRICAL SPHERICAL

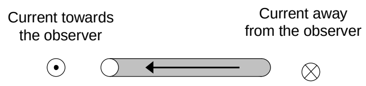
Field Vectors

- The same E-field can be described using different coordinate systems.
- THIS FIELD IS INDEPENDENT OF THE COORDINATE SYSTEM!!!

E-vector	Coordinates	Range of Coordinates
$\vec{E} = \vec{a}_x E_x + \vec{a}_y E_y + \vec{a}_z E_z$	cartesian (x, y, z)	$-\infty < x < \infty$ $-\infty < y < \infty$ $-\infty < z < \infty$
$\vec{E} = \vec{a}_r E_r + \vec{a}_\theta E_\theta + \vec{a}_z E_z$	cylindrical (r, θ, z)	$0 \le r < \infty$ $0 \le \phi < 2\pi$ $-\infty < z < \infty$
$\vec{E} = \vec{a}_r E_r + \vec{a}_\theta E_\theta + \vec{a}_\phi E_\phi$	spherical (r, θ, ϕ)	$0 \le r < \infty$ $0 \le \theta \le \pi$ $0 \le \phi < 2\pi$

Drawing current directed at right angles to the page.

- The following is used for representing current flowing towards or away from the observer.

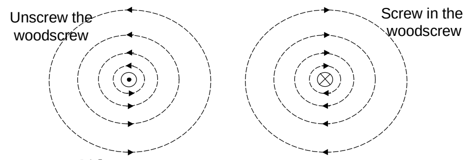


Memory Aid:

Think of a dart,
with the POINT (arrowhead) travelling TOWARDS you
and the TAIL (feather) travelling AWAY from you.

The grip rule

- Now draw the field around a current carrying conductor using the RIGHT-HAND THREAD rule.



Memory Aid:

Grip your right hand around the conductor
with your thumb in the same direction as the conductor.
Your 4 fingers now show the direction of the magnetic field.