Electrostatics & Magnetostatics

22.3MB1

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Capacitance

- Capacitance is the ratio of charge to electric potential difference.

$$C = \frac{Q}{V} \quad (F)$$

- C = capacitance (Farads)
- Q = capacitor charge (Coulombs)
- V = potential difference (Volts)

Three steps to calculating capacitance

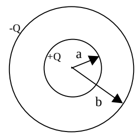
STEP1 Apply Gauss's law to find the flux density and the electric field distribution. (**D** = ε**E**)	$\oint_S \vec{D}.d\vec{S} = Q_{enc} \quad (C)$
STEP2 Apply the potential law to find the potential difference between the conductors which form the structure of the capacitor.	$V_{AB} = -\int_B^A \vec{E}.d\vec{l} \quad (V)$
STEP3 Apply Q=CV or an equivalent. As in the case of a line conductor (λL=CV). This will give capacitance per unit length.	$C = \frac{Q}{V_{AB}} \quad (F)$ $C = \frac{\lambda}{V_{AB}} \quad (F/m)$

Capacitance of three simple structures

$$C = \frac{Q}{V} \quad (F)$$

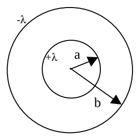
- Coaxial spheres

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad (V)$$



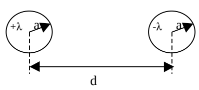
- Coaxial cylinders

$$V_{AB} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right) \quad (V)$$

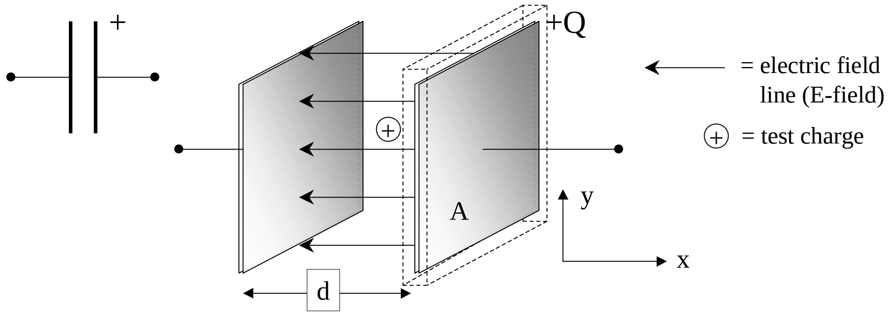


- Twin conductors

$$V_{AB} = \frac{\lambda}{\pi\varepsilon_0}\ln\left(\frac{d-a}{a}\right) \quad (V)$$



Electrostatic analysis of a parallel plate capacitor
(assume no fringing fields)



flux density	field intensity	potential difference	capacitance	
$D_x = \frac{Q}{A}$ ⇨	$E_x = \frac{Q}{\varepsilon A}$ ⇨	$V = \frac{Qd}{\varepsilon A}$ ⇨	$C = \frac{Q}{V}$ ⇨	$C = \frac{\varepsilon A}{d}$
(C/m^2)	(V/m)	(V)	(F)	

Flux Linkage

- Inductance is calculated using the notion of **flux linkage**.

$$L = \frac{\Lambda}{I} \quad (H)$$

- Λ = flux linkage (Weber-turns)
- L = inductance (Henries)
- I = current (Amperes)

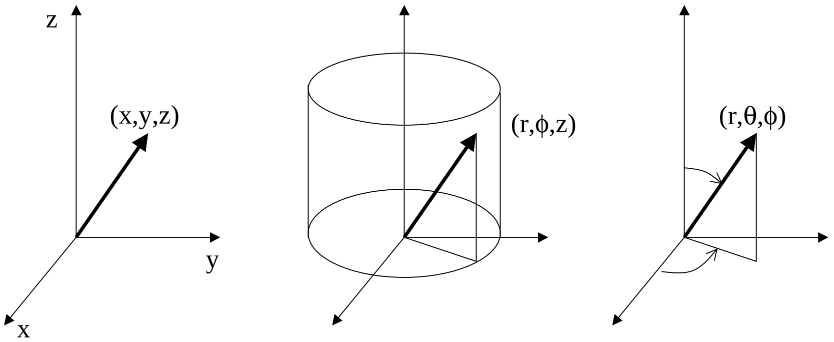
- **For self inductance cases:-**
  - Flux linkage is the linkage of magnetic flux from one magnetic circuit to itself.
- **For mutual inductance cases:-**
  - Flux linkage is the linkage of magnetic flux from one magnetic circuit to another.

Three steps to calculating inductance

STEP1 Apply Ampere's law to determine the magnetic field distribution around the circuit. Then find the flux density.	$\oint_l \vec{H} . d\vec{l} = I_{enc} \quad (A)$ $\vec{B} = \mu_0 \mu_r \vec{H} \quad (Wb/m^2)$
STEP2 Apply the flux law to find the total flux passing through the circuit. Then determine the flux linkage, (shown is for a N-turn coil).	$\Phi = \int_S \vec{B}.d\vec{S} \quad (Wb)$ $\Lambda = N\Phi$ (Wb-turns)
STEP3 Apply $\Lambda = LI$ to find the inductance.	$L = \frac{\Lambda}{I} \quad (H)$

Co-ordinate Systems

- Using the appropriate coordinate system can simplify the solution to a problem. In selecting the correct one you should be looking to find the natural symmetry of the problem itself.



CARTESIAN CYLINDRICAL SPHERICAL

Field Vectors

- The same E-field can be described using different coordinate systems.
- THIS FIELD IS INDEPENDENT OF THE COORDINATE SYSTEM!!!

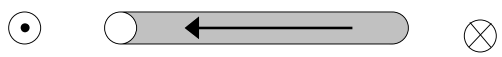
E-vector	Coordinates	Range of Coordinates
$\vec{E} = \vec{a}_x E_x + \vec{a}_y E_y + \vec{a}_z E_z$	cartesian (x, y, z)	$-\infty < x < \infty$ $-\infty < y < \infty$ $-\infty < z < \infty$
$\vec{E} = \vec{a}_r E_r + \vec{a}_\theta E_\theta + \vec{a}_z E_z$	cylindrical (r, θ, z)	$0 \le r < \infty$ $0 \le \phi < 2\pi$ $-\infty < z < \infty$
$\vec{E} = \vec{a}_r E_r + \vec{a}_\theta E_\theta + \vec{a}_\phi E_\phi$	spherical (r, θ, ϕ)	$0 \le r < \infty$ $0 \le \theta \le \pi$ $0 \le \phi < 2\pi$

Drawing current directed at right angles to the page.

- The following is used for representing current flowing towards or away from the observer.

Current towards the observer

Current away from the observer

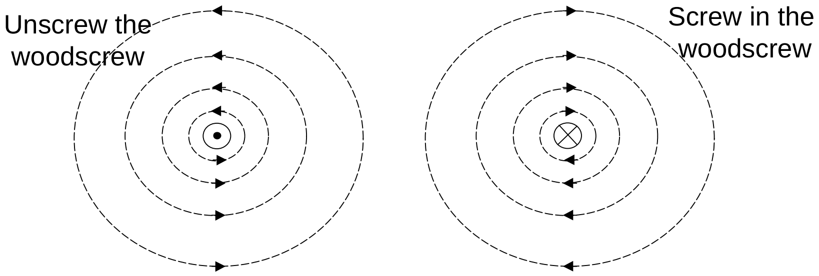


Memory Aid:

Think of a dart,
with the POINT (arrowhead) travelling TOWARDS you
and the TAIL (feather) travelling AWAY from you.

The grip rule

- Now draw the field around a current carrying conductor using the RIGHT-HAND THREAD rule.



Memory Aid:

Grip your right hand around the conductor
with your thumb in the same direction as the conductor.
Your 4 fingers now show the direction of the magnetic field.