Question April 2000

(a)
Maxwell's two 'curl' equations are as follows:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{dt}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{dt}$$

Make use of these equations to derive the following wave equation for the electric field component of a time-changing electromagnetic wave propagating in a uniform, linear, isotropic, non-magnetic region of conductivity σ Sm^{-1}, permittivity, ε Fm^{-1}, and permeability μ Hm^{-1}:

$$\nabla^2 \mathbf{E} = \mu_o \sigma \frac{\partial \mathbf{E}}{\partial t} + \mu_o \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

If the time variation of E is sinusoidal show that the above equation may be expressed as:

$$\nabla^2 \mathbf{E} - \gamma^2 \mathbf{E} = 0$$

where

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

If the time variation of **E** is restricted to variation along the z-axis only, show that the above wave equation may be further simplified to

Show that $\mathbf{E} = E_o e^{-\gamma z}$ is a valid solution to this simplified wave equation.

Writing γ as $\gamma = \alpha + j\beta$ obtain expressions for α and β for the extreme cases of

(i) a 'perfect dielectric' and
(ii) a 'good conductor'

Consider the following four uniform regions with regard to a 1MHz plane TEM wave. decide for which case(s) a 'good conductor' or a 'perfect dielectric' approximation may apply, and for such case(s) calculate the following parameters:

(a) attenuation constant α (dB m^{-1})
(b) phase constant β (degree m^{-1})