22.4UC2

Assignment 3



This assignment is due at my office, office 3.20 at the Department of Computing and Electrical Engineering, on Friday, March 7, before 4:30 pm. Complete the details below and staple this sheet to the front of your solutions. Please *underline* your family name.

Name: _____ Student number: ____

LECTURER: YVAN PETILLOT

1. CD's are examined for scratches. 150 CD's give rise to the following data:

Number of scratches per item	0	1	2	3	4	5	6
Frequency	11	45	38	35	14	5	2

Let x be the random variable 'number of scratches on the surface of a randomly chosen item', and suppose that x has a Poisson distribution with parameter λ .

- (a) Find an estimator for λ and calculate the estimate for the above data.
- (b) Compute the estimated standard error and calculate the standard error of the data above.
- 2. Let x denote the proportion of alloted time that a randomly selected student works on a certain assignment. Suppose that the distribution of x has the following density function:

$$\begin{cases} (\theta+1)x^{\theta} & \text{if } 0 \le x \le 1\\ 0 & \text{else} \end{cases}$$

with parameter $\theta > -1$. Find the maximum likelihood estimator of θ and then compute an estimate for it with the following data:

$$0.91 \quad 0.89 \quad 0.67 \quad 0.45 \quad 0.98 \quad 0.81 \quad 0.70 \quad 0.78 \quad 0.95 \quad 0.86$$

3. The $(1-\alpha)100\%$ confidence interval for the mean of a normal population with known standard deviation σ is

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where \bar{x} is the sample mean.

- (a) How much must the sample size n be increased if the length of the confidence interval is to be halved? Justify your answer.
- (b) If the sample size is increased by a factor of 25, what effect will this have on the length of the interval? Justify your answer.
- 4. A human resources officer in a large company wants to survey what proportion of the work force is aware of her position.
 - (a) What sample size is necessary if the 95% confidence interval for p is to have length at most 0.1, irrespective of p.
 - (b) If the officer has strong reasons to believe that p = 2/3, what sample size should be recommended?

5. For the following sets of data calculate a 99% confidence interval for the difference between the true average.

$sample \dots$	size	mean	standard dev.
		26.99	4.89
$\operatorname{set} 2$	74	35.76	6.43

- 6. Consider the data for EU Council of Ministers given on slides Unit 6-14 and Unit 6-15.
 - (a) For the *combined list* of all 27 countries calculate the least squares regression line. For each country calculate the error between predicted number of seats and observed number of seats.
 - (This question is not part of the assignment, but can you think of reasons why some data (like that of Germany) are so far off the least squares regression line?)
 - (b) Calculate the correlation factor for this set of data.
 - (c) Find a 95% confidence interval for the true slope of the least squares regression line. Give an interpretation of this confidence interval for this set of data.