22.4UC2 Revision Exercises — Week 9

Pick some of the exercises below for your tutorial. Sorry, but I didn't have time to calculate solutions.

Exercise 1 Let x_i, $0 \leq i \leq n$ be a random sample of n observations of a random variable x with density function

$$\begin{cases} \frac{xe^{-x/\beta}}{\beta^2} & \text{if } x \geq 0 \\ 0 & \text{else.} \end{cases}$$

(a). Find the maximum likelihood estimator $\hat{\beta}$ for β.

(b). Find $E(\hat{\beta})$ and $\sigma_{\hat{\beta}}$.

Exercise 2 The computer centre of a university wants to estimate the variability of the number of processed computer jobs per day. A random sample of 18 days during a period of 3 months is selected and the number of processed jobs per day is counted. It is found that the random variable x number of processed computer jobs per day has sample mean $\bar{x} = 1130$ and $s = 160$. What is a 99% confidence interval for the true variability of jobs per day? What kind of assumptions are necessary to draw this conclusion?

Exercise 3 A machine is to produce screws with mean length of 1cm. Screws too short or too long to meet the customers specifications are rejected. To avoid producing too many rejects the screws produced by the machine are sampled on a regular basis as a check to determine whether the machine is still operating properly.
50 screws have been sampled, and $\bar{x} = 1.02$cm and the sample mean is $s = 0.04$cm. Is there evidence, using level of significance 0.01, that the mean length of screws produced by the machine is *not* 1cm?

Exercise 4 A regional bus company implements a new staff training program to reduce tardiness of buses. At randomly selected local stops buses are observed before and after the program was implemented, and the following data was collected:

- Before the training program 30 stops were observed. The average bus was 5.25 minutes late, with sample error 1.88 minutes.
- After the training program another 35 bus stops were observed. Now the buses were on average 2.37 minutes late, with sample error 1.34 minutes.

Use a significance level of 0.02 to test whether staff training resulted in an improvement of service.

Exercise 5 The US Federal Communications Commission (FCC) specifies that radiated electro magnetic emissions from digital devices are to be measured in an open-field test site. In order to verify test-site acceptability the site attenuation (transmission loss from the input of one half-wave dipole to the output of another when both dipoles are positioned over the ground plane) must be evaluated. A study conducted at a test site in Fort Collins, Colorado, yielded the following data on site attenuations (in decibels) and transmission frequency (in megahertz) for dipoles at a distance of 3 meters. (The data are from Bennett, W. W., An Error Analysis of the FCC Site-Attenuation Approximation, *IEEE Transaction on Electromagnetic Compatibility*, EMC-27, p. 113.)

Transmission frequency	Site attenuation
50	11.5
100	15.8
200	18.2
300	22.6
400	26.2
500	27.1
600	29.5
700	30.7
800	31.3
900	32.6
1000	34.9

(a). Draw a scatter diagram for this set of data. Interpret the diagram.

(b). Find the correlation factor for this set of data.

(c). Find the least squares line relating site attenuation depending on the transmission frequency.

(d). Calculate a 95% confidence interval for the true slope of the least squares line.