22.4UC2 **Tutorial Exercises — Week 3**

Pick some of the exercises below for your tutorial.

Exercise 1 Recall from calculus that $(a+b)^n = \sum_{i=0}^{n} \binom{n}{k} a^k b^{n-k}$.

(a). Use this fact to show that for a binomial probability distribution of length n and success probability p, $\sum_{\text{all } x} p(x) = 1$.

(b). Prove that if x has binomial probability distribution with parameters n and p then $E(x) = np$.

[*Hint:* Write out the definition of $E(x)$, factor out n and p, to get $E(x) = np \sum \ldots$, and recognise the remaining sum as the probabilities of a Binomial random variable for some n' and p'.]

Exercise 2 On New Year's Eve the police makes random controls of car drivers. Suppose they find that about 20% of the drivers have consumed alcohol. What is the probability that among five checked drivers at most 3 have drunk? *[99.33%.]*

Exercise 3 Say the probability of passing the road test (for a driver's licence) is 75%, without changing with every try. What are the probabilities that a young drier will pass the test on the second (or fourth) attempt? *[18.75%, 1.17%.]*

Exercise 4 Among 100 applicants to university only 60 are actually qualified. If a university selects randomly 6 students for interviews, what is the probability that only 3 of them are qualified? (Do two calculations, using the binomial and the hyper-geometric distribution.) *[27.65%, 28.36%.]*

Exercise 5 Burning tiles is a difficult task. Suppose that certain tiles are known to have 5 defects per 10 square feet (with Poisson distribution). What is the probability that a randomly selected tile of 1 square foot is of high quality, i.e., has no defect? *[60%.]*

Exercise 6 What is the probability that in a room of x people at least two of them are born on the same day? How many people are needed so that this probability is greater than 50%? What is the probability that among 12 people 2 have their birthday on the same day? *[*$1 - \frac{365 \cdot 364 \cdots (365-x+1)}{365^x}$*, 23 (then the probability is 50.73%), 16.70%.]*

Exercise 7 A regional airport has about 56.4 takeoffs per day. Suppose that the random variable takeoffs has a Poisson distribution. What is the probability that there will be at least 2 takeoffs within one hour? What is the probability that there is at least 1 hour between two takeoffs? How do these values change if takeoffs happen only between 8 am and 8 pm? (Frankfurt airport has about 1200 takeoffs per day.) *[68.05%, 65.34%, 94.82%, 80.83%.]*