Module 22.5SD2
Tutorial 1 : Fourier Transform

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1 Problem 1

The sequence $f(x, y)$ was defined to be periodic with a period of $N_1 \times N_2$ if

$$f(x, y) = f(x + N_1, y) = f(x, y + N_2) \quad for\ all\ x, y$$

More generally defined the condition is

$$f(x, y) = f(x + N_{11}, y + N_{21}) = f(x + N_{12}, y + N_{22})$$

Show the number of independent samples in the period is

$$N_{11}N_{22} - N_{12}N_{21}$$

using the figure below for reference

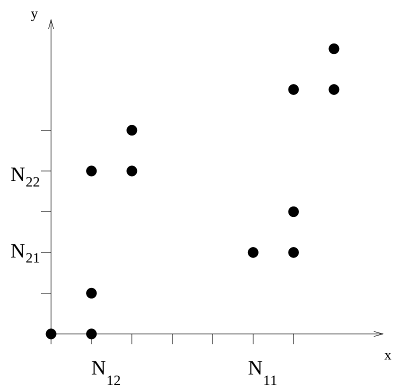

Figure 1: Reference periods for calculus

2 Problem 2

A simple edge detector could have a convolution mask of the form

$$[1 - 1]$$

Determine the magnitude of the frequency response of this operator and plot it.

3 Problem 3

Evaluate the DFT of a 2×2 image using the matrix expression

$$T = AFA$$

and show that the result is equivalent to calculating the DFT from

$$T(u, v) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) \exp[-j2\pi(ux + vy)/N]$$

4 Problem 4

Discretizing a signal corresponds to periodizing its spectrum. What are the implications of this theorem on:

1. Sampling frequency requirements for a signal with a band-limited spectrum in the band[-B B].
2. If the sampling frequency requirements are met, describe how you can recover the original signal from the discretized one.
3. What happens if they are not met