Distributed Subchannel Allocation for Interference Mitigation in OFDMA Femtocells: A Utility-Based Learning Approach

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***Abstract*—Both orthogonal frequency-division multiple access (OFDMA) and femtocell are promising technologies providing subscribers with better services. However, due to the ad hoc nature of femtocells, there is a great challenge to mitigate interference, which may seriously compromise the benefits promised by this novel network architecture. This paper investigates the distributed subchannel allocation (DSA) for cotier interference mitigation in OFDMA-based femtocells, where the femtocells and macrocell transmit on orthogonal subchannels. Particularly, to intuitively study system performance, we formulate this problem as a noncooperative rate maximization game where the utility of each player or femtocell access point is its capacity instead of the incoming interference. Unfortunately, the uncertainty of the existence of the Nash equilibrium for this game makes it difficult to design efficient distributed schemes. To address this issue, we introduce a state space to reflect players' desire for new strategies and then devise a utility-based learning model that requires no information exchange between different players. Utilizing this model, a utility-based DSA algorithm is developed. Moreover, it is analytically shown that the Pareto-optimal solution can be achieved with our proposed algorithm, and as a result, the overall capacity can be efficiently improved, and the system interference can be efficiently mitigated. Finally, simulation results verify the validity of our analysis and demonstrate that our scheme performs comparably or even better compared with the existing strategies, which require information exchange among different femtocells.**

***Index Terms*—Distributed subchannel allocation, femtocells, interference mitigation, orthogonal frequency-division multiple access (OFDMA), Pareto optimality.**

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I. Introduction

FEMTOCELL is a promising technology to fulfill the explosive demand of high-data-rate services and the requirement of ubiquitous access [1]–[3]. To be specific, in contrast to conventional macrocell base stations (BSs), femtocell access points (FAPs) or the so-called home BSs are lower-power, short-range, plug-and-play small BSs that are installed and managed by customers in residential areas and small offices. With the help of this novel technology, more users can share the same spectrum resource by accessing different femtocells in different areas. On the other hand, users in poor-indoor-coverage regions or dead zones can achieve better performance by deploying additional FAPs. Nevertheless, to exploit the benefits promised by femtocells, it is greatly necessary to mitigate both cotier and cross-tier interference, which emerge when the same spectrum is allocated to different cells [4]. Particularly, cotier interference refers to the interference among femtocells or macrocells, whereas cross-tier interference is the interference between macrocells and femtocells.

Orthogonal frequency-division multiple access (OFDMA) technology can exploit channel variations in both frequency and time domains by dividing the available spectrum into orthogonal subchannels. Therefore, compared with other multiple-access schemes, e.g., code-division multiple access, it provides more flexibility in interference mitigation for femtocell networks [5]. In OFDMA femtocell networks, the dedicated-channel deployment (or orthogonal channel assignment) can be used to cope with cross-tier interference, i.e., femtocells would be allocated a fraction of subchannels, whereas the macrocell would be allocated another fraction [4]–[6]. Although this strategy is suboptimal from the spectral efficiency standpoint, it is a relative simple solution to avoid cross-interference and has been considered as one option for enhanced intercell interference coordination in LTE Release 10 specifications [7].

Although cross-tier interference can be avoided with dedicated-channel deployment, to mitigate cotier interference and further improve the system capacity, efficient subchannel allocation schemes for femtocells are necessary. Meanwhile, because of the uncertainty in the number and positions of the femtocells, it is not viable to perform a centralized resource allocation [8]. Therefore, in this paper, we study the issue of distributed subchannel allocation (DSA) in OFDMA femtocell networks, where there is no cross-tier interference being considered. To intuitively study the system performance, we formulate this problem as a noncooperative rate maximization game (NRMG), where the utility of each femtocell is its capacity. However, it has been proved that there is no guarantee that the Nash equilibrium (NE) for NRMG always exists, which makes it more difficult to design distributed strategies to achieve stable and efficient solutions. To get around this difficulty, previous studies generally change the player's utility function, which may cause the loss of system capacity. In this paper, motivated by the studies of the utility-based learning model, which is appropriate for studying multiagent systems [9]–[12], we have introduced an additional state space in NRMG and then proposed a utility-based DSA algorithm (UDSA). Our major contributions can be summarized as follows.

- We introduce a new degree of freedom to address the inherent problem in NRMG, and it is analytically shown that the proposed UDSA converges to the Pareto-optimal solution.[1]
- We propose a completely DSA strategy, where there is no channel state information (CSI) exchange among the autonomous agents, and moreover, each agent does not need to know the strategies adopted by the others. Therefore, this strategy is also appropriate for other noninfrastructure systems, e.g., ad hoc networks and no network-assisted device-to-device systems.
- Simulation results show that the proposed algorithm could be a good candidate for OFDMA femtocells because of its adaptability in different interference environments, i.e., the relatively higher overall capacity can always be achieved.

The remainder of this paper is organized as follows. In Section II, we summarize the related work. The description of the system model and formulation of the DSA are given in Section III. Section IV briefly introduces the utility-based learning model, proposes a utility-based decentralized subchannel allocation algorithm, and analyzes the performance of the developed algorithm. Numerical and simulation results are presented and analyzed in Section V, and concluding remarks are given in Section VI.

II. Related Work

When dedicated-channel deployment is adopted in an OFDMA femtocell network, cross-tier interference is avoided, but cotier interference still needs to be considered. It should be noted that this intercell interference can be mitigated with interference cancelation [13] and interference alignment [14]. However, due to the heavy communication overhead of these techniques, dynamic frequency allocation schemes are recommended for femtocells [15]. By constructing the interference graph for a femtocell network, the femtocell grouping-based and greedy-based resource allocation strategies were investigated in [16] and [17], respectively. Energy-efficient resource allocation for cognitive radio femtocell networks was studied in [18], where cognitive BSs sold the spectrum bought from the primary networks to FAPs. This problem was formulated as a Stackelberg game, and a gradient-based iteration algorithm was devised to achieve the equilibrium state.

In all of the aforementioned works [16]–[18], the central controller is necessary for femtocell management, which would require significant signaling overhead and cause congestion in the backhaul network[2] [4]. For this reason, some researchers begin to turn their attention to distributed interference mitigation in OFDMA femtocells [15], [20]–[25]. To be specific, the dedicated subchannels were allocated to femtocells [or femtocell user (FU)] with a random manner in [20] and [21]. In [15], [22], and [23], when choosing from the available subchannels, each FAP or FU just considered the existing interference on each subchannel (i.e., incoming interference), which resulted in severe interference within the network and reduced the overall capacity of femtocells. To further improve the overall capacity, not only incoming interference but also outgoing interference (i.e., the interference caused to existing femtocells) was considered in [24] and [25]. Particularly, the proposed autonomous component carrier selection and its improved version could achieve better aggregate performance at the cost of higher signaling overhead. In comparison, our algorithm can be employed in a distributed fashion without information exchange between FAPs and, meanwhile, improve the sum capacity of femtocells.

As a mathematical tool for analyzing conflict and cooperation between autonomous agents, game theory has been widely used to design interference mitigation schemes in multicell OFDMA systems [26]–[28]. We would like to note that these distributed strategies can be easily extended to OFDMA femtocells. Specifically, rate maximization with power pricing game was formulated in [26], and an NE could be achieved with the proposed algorithm. Moreover, in [26], all cells operated in full load, i.e., all subchannels were utilized. However, in general cases, BSs would choose subchannels according to traffic load, and then, there is no guarantee that the NE for NRMG always exists [27]. As pointed out in [28], due to the nonexistence of NEs, it is a challenge to devise a game-based learning algorithm to achieve stable solutions. To get around this difficulty, both [27] and [28] formulated the DSA as a potential game, where there is a wide class of learning algorithms converging to a pure NE, e.g., gradient play, fictions play, and joint strategy fictitious play [29].

However, there are some inherent shortcomings in the formulated potential game, which would affect the performance of the proposed algorithm. For instance, to formulate a potential game, the corresponding utility function should be properly designed. As a result, there may be no obvious relationship between the utility and capacity. In other words, an increase in total utility is not equivalent to a higher overall capacity [27], [28]. Moreover, the efficiency of the achieved NE cannot be guaranteed, and the information exchange among players requires additional overheads. In this paper, we formulate the distributed interference mitigation as NRMG and develop a distributed algorithm without information exchange between

[1]Although directly minimizing the interference is not the aim of this work, the system interference can be efficiently mitigated if the Pareto-optimal solution can be achieved, and the overall capacity can be efficiently improved.

[2]In a femtocell network, FAPs can transmit limited signaling data over the backhaul network via residential wireline broadband access lnks, e.g., digital subscriber lines [19].

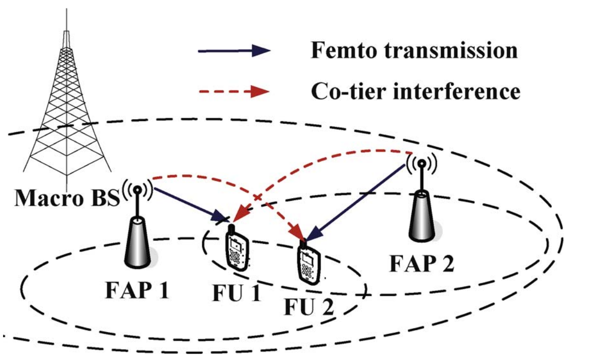


Fig. 1. Downlink cotier interference in an OFDMA femtocell network configured in CAM, where femtocells are sharing the dedicated subchannels to avoid cross-tier interference.

different players. Additionally, we have proved the Pareto optimality of the solution achieved by our proposed algorithm.

III. System Model and Problem Formulation

Let us consider a femtocell network, where FAPs are deployed in random locations and configured in closed access mode (CAM).[3] Particularly, there are in total N femtocells sharing the dedicated spectrum consisting of K orthogonal subchannels, each of which has bandwidth B_0. Therefore, the cross-tier interference in this network is avoided. For a given slot, if different FAPs (or FUs) transmit on the same subchannel, downlink (or uplink) cotier interference occurs. For analysis simplicity, we consider that there is only one FU communicating with the FAP in each time slot, which has been assumed in previous studies due to the opportunistic scheduling operation [18], [30]. In this paper, we mainly study the interference mitigation for downlink communication. However, under the given assumptions, the proposed scheme is also suitable for the uplink case. As shown in Fig. 1, without loss of generality, we label the FU subscribing to FAP n by FU n. Moreover, let us assume that each FAP n chooses K_n subchannels to be assigned for the data transmission to FU n, where $0 < K_n < K, \forall\, n \in \mathcal{N}$.[4] Similar to previous studies [27], [28], [31], we only consider subchannel allocation for femtocells. However, optimizing the number of subchannels for each FU is out of the scope of this paper.

Let us denote the channel gain matrix by $\mathbf{G} \in \mathbb{R}^{N\times N\times K}$, where element $g_{n,m}^k$ represents the channel gain between FAP n and FU m on subchannel k. Similar to many of the previous studies [8], [32], [33] that focus on femtocell networks, we just consider path loss and shadowing fading for analysis simplicity. In particular, let $d_{n,m}$ denote the distance between FAP n and FU m, and $g_{n,m}^k$ in decibels is [32]

$$\begin{cases} -28 - 35\log_{10}(d_{n,m}) - \psi, & n = m \\ -38.5 - 20\log_{10}(d_{n,m}) - L_{\text{wall}} - \psi, & n \neq m \end{cases} \tag{1}$$

where L_{wall} and ψ are the wall penetration loss and Gauss-distributed random variable with mean zero and variance σ_ψ^2, respectively. It is assumed in this paper that each FAP n only knows the channel gains between itself and its subscriber, i.e., $g_{n,n}^k$, $k \in \{1, 2, \dots, K\}$. Compared with the assumptions in [27] and [28], we dispense with a large signaling overhead to exchange CSI between any two different femtocells, i.e., $g_{m,n}^k$ and $g_{n,m}^k$, where $\forall\, n, m \in \{1, 2, \dots, N\}, m \neq n$, and $\forall\, k \in \{1, 2, \dots, K\}$.

Let $\mathcal{S}_n$ be the set of subchannels assigned to FU n, i.e., $\mathcal{S}_n = \{s_1, s_2, \dots, s_{K_n}\}$, where $s_k \in \{1, 2, \dots, K\}$ is the index of the kth subchannel allocated to FU n. Moreover, we denote the transmit power set of FAP n on $\mathcal{S}_n$ by $\mathcal{P}_n$, i.e.,

$$\mathcal{P}_n = \left\{p_n^{s_1}, p_n^{s_2}, \dots, p_n^{s_{K_n}}\right\}, \quad \sum_{k=1}^{K_n} p_n^{s_k} \leq p_{n,\max} \tag{2}$$

where $p_n^{s_k}$ represents the transmit power of FAP n on subchannel s_k, and $P_{n,\max}$ is the power limit. To facilitate analysis, we do not consider power control and assume that $\mathcal{P}_n (\forall n \in \{1, 2, \dots, N\})$ is arbitrary, reasonable, and fixed during the underlying operational period. Meanwhile, it should be noted that both the uniform distribution mechanism (i.e., $p_n^{s_k} = p_{n,\max}/K_n, \forall\, k \in \{1, 2, \dots, K_n\}$ and $\forall\, n \in \{1, 2, \dots, N\}$) used in [34] and the simple power management schemes proposed in [28] and [31] can be adopted to determine the set $\mathcal{P}_n$.

We model the additive noise as a zero-mean Gaussian random variable. For FAP n with subchannel $k \in \mathcal{S}_n$, the signal-to-interference-plus-noise ratio (SINR) can be expressed as

$$\begin{aligned} \gamma_n^k &= \frac{p_n^k g_{n,n}^k}{I_n^k + B_0 N_0} \\ &= \frac{p_n^k g_{n,n}^k}{\sum_{m=1, m\neq n}^{N} \sum_{l \in \mathcal{S}_m \cap \{k\}} p_m^l g_{m,n}^l + B_0 N_0} \end{aligned} \tag{3}$$

where I_n^k represents the cotier interference caused to FAP n on subchannel k, B_0 is the bandwidth of each subchannel, and N_0 is the noise power density. We use Shannon capacity to model the maximal achievable rate. Then, the capacity of femtocell n can be expressed as

$$R_n = \sum_{k\in\mathcal{S}_n} R_n^k = \sum_{k\in\mathcal{S}_n} B_0 \log_2\left(1 + \gamma_n^k\right). \tag{4}$$

To maximize the capacity of the femtocell, each FAP has to choose the subchannels with lower interference. Specifically, according to (3) and (4), it can be seen that FAP n should make its decision based on other FAPs' actions, and, in return, the choice of FAP n will also affect other FAPs' decisions. To study the strategic interaction among these autonomous FAPs, game theory is considered as a potentially effective tool. In particular, we can formulate this subchannel selection problem as NRMG, which is defined as follows.

1) Definition 1—NRMG: NRMG can be represented by the tuple $\mathcal{G} = \Gamma(\mathcal{N}, (\mathbb{S}_n)_{n\in\mathcal{N}}, (U_n)_{n\in\mathcal{N}})$, where $\mathcal{N} = \{1, 2, \dots, N\}$ is the set of players corresponding to N FAPs. For each player

[3] Three access modes have been defined for a femtocell network: open access mode, CAM, and hybrid access mode, respectively [4].

[4] Note that when $K_n = 0$ or $K_n = N$, there is no subchannel selection for FAP n. In this case, the FAP n will not be included in our consideration.

n, its strategy space $\mathbb{S}_n$ is the available sets of subchannels, which can be expressed as

$$\mathbb{S}_n = \{\mathcal{S}_n | \mathcal{S}_n \subset \{1, 2, \ldots, K\},\ |\mathcal{S}_n| = K_n\} \tag{5}$$

where $|\cdot|$ denotes the cardinality of a set. Given a strategy profile $(\mathcal{S}_n)_{n\in\mathcal{N}} = (\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N) \in (\mathbb{S}_n)_{n\in\mathcal{N}}$, the utility function of each player n is

$$U_n\left((\mathcal{S}_n)_{n\in\mathbf{N}}\right) = \sum_{k\in\mathcal{S}_n} B_0 \log_2\left(1 + \frac{p_n^k g_{n,n}^k}{I_n^k(\mathcal{S}_{-n}) + B_0 N_0}\right)\ \forall n \in \mathcal{N} \tag{6}$$

where $\mathcal{S}_{-n} = (\mathcal{S}_1, \ldots, \mathcal{S}_{n-1}, \mathcal{S}_{n+1}, \ldots, \mathcal{S}_N)$ is the strategy profile of all players other than player n, and moreover, $I_n^k(\mathcal{S}_{-n})$ represents the interference caused to player n by other players. Additionally, we note that the size of the strategy space of each player n is $|\mathbb{S}_n| = C_K^{K_n} = K!/K_n!(K - K_n)!$, where $K!$, $K_n!$, and $(K - K_n)!$ denote the factorials of K, K_n, and $K - K_n$, respectively.

According to Definition 1, the terms FAP and player will be used interchangeably hereafter. Because of the conflicts among players and the absence of central authority, the social efficiency of a multiagent system will always be reduced, which is termed as price of anarchy. In this paper, we aim at devising an efficient distributed scheme to improve the social welfare, i.e., the sum capacity of femtocells $R = \sum_{n=1}^{N} R_n$. To this end, it is essential to design local learning models, with which players can update their strategies according to the environment. As shown in [35], there are some distributed learning models having been derived based on NE, which is a standard solution standing for the equilibrium state of a noncooperative game. In this light, we will next study the NE for NRMG before designing the DSA strategy.

Definition 2—NE: For NRMG, $\mathcal{G} = \Gamma(\mathcal{N}, (\mathbb{S}_n)_{n\in\mathcal{N}}, (U_n)_{n\in\mathcal{N}})$, if a profile

$$\mathcal{S}^* = (\mathcal{S}_1^*, \mathcal{S}_2^*, \ldots, \mathcal{S}_N^*) \tag{7}$$

in the strategy space $(\mathbb{S}_n)_{n\in\mathcal{N}}$ is an NE, no player can unilaterally improve its own utility by choosing a different strategy. This means that

$$U_n\left(\mathcal{S}_n^*, \mathcal{S}_{-n}^*\right) \geq U_n\left(\mathcal{S}_n, \mathcal{S}_{-n}^*\right)\ \forall \mathcal{S}_n \in \mathbb{S}_n,\ \forall n \in \mathcal{N} \tag{8}$$

where $\mathcal{S}_{-n}^* = (\mathcal{S}_1^*, \ldots, \mathcal{S}_{n-1}^*, \mathcal{S}_{n+1}^*, \ldots, \mathcal{S}_N^*)$.

Unfortunately, the existence of the NE for NRMG cannot be guaranteed. To show this, here, we consider a "toy" two-player case where $K = 2$, $N = 2$, and $K_1 = K_2 = 1$. Then, for each player $n \in \{1, 2\}$, we have $\mathcal{S}_n = \{s_n\}$, $s_n \in \{1, 2\}$, and

$$\arg\max_{s_n} U_n(s_1, s_2) = \arg\max_{s_n} \gamma_n^{s_n}(s_1, s_2). \tag{9}$$

In this case, the formulated NRMG is identical to the SINR maximization game introduced in [27]. Based on the numerical example presented in [27, Tab. I],[5] we note that the SINR maximization game may have no NE. Hence, there is also no guarantee that NRMG always admits an NE. On top of this conclusion, it brings us a great challenge of designing an efficient DSA scheme for the formulated problem. The issue for efficient algorithm design will be addressed in detail in the following section.

IV. Distributed Subchannel Allocation Algorithm

Here, a quick overview of utility-based learning models is given, and then, the algorithm UDSA is developed. After that, the performance of the proposed scheme is mathematically analyzed.

A. Utility-Based Learning Models

For NRMG, $\Gamma(\mathcal{N}, (\mathbb{S}_n)_{n\in\mathcal{N}}, (U_n)_{n\in\mathcal{N}})$, at each decision time $t \in \{0, 1, 2, \ldots\}$, each player n would choose a strategy (or an action) $\mathcal{S}_n(t) \in \mathbb{S}_n$ and receive the corresponding utility $U_n((\mathcal{S}_n(t))_{n\in\mathcal{N}})$. For individual players, their strategies will be determined by the observations from times $\{0, 1, \ldots, t-1\}$ and the predefined decision rule, which is referred to as the learning model. In other words, different learning models are specified by both the assumptions on available information and the rules for choosing strategies. For instance, in best-response dynamic, given the strategies of other players, each player responds by choosing the strategy that maximizes its own utility.

In fact, there is a kind of learning models that are said to be utility-based or payoff-based [9]. In these models, it is assumed that each player can only access the history of its own actions and utilities, and players have to make decisions based on the limited information. For this reason, such models are considered to be more applicable for studying the multiagent systems, where the information exchange between different agents is strictly restrained. However, due to the limited available information, both the convergence and efficiency of this model are often unpredictable. Recently, Li and Marden began to address this issue in their studies [10]–[12], and they demonstrated that it is critical to introduce an additional degree of freedom to the formulated game or to the proposed learning algorithm.

B. UDSA

By introducing a state to reflect the player's desire for new strategies, we will devise a utility-based learning model here. Then, our algorithm UDSA will be developed based on the proposed learning model. The details are given as follows.

Here, to devise the utility-based learning model, we reconstruct NRMG by introducing an additional state to each player and term it as personality. Particularly, players can be divided into two groups based on their personality: the conservative and the radical. Let us denote the personality of conservatives and radicals by c and r, respectively. According to the player's desire for new strategies, the difference between these two groups of players will be presented later. Then, at each decision moment t, every player n can be described by

[5] Note that there is a minor notation difference between NRMG and that formulated in [27]. Particulary, $g_{n,m}$ denotes the channel power gain from FAP n to FU m in this paper, and $h_{n,m}^2$ denotes the channel power gain from BS m to user n in [27].

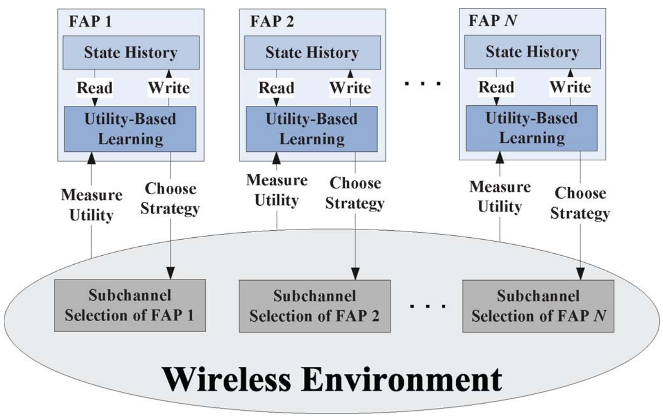


Fig. 2. Diagram of the utility-based learning model.

a triplet $\mathbb{L}_n(t) = (\mathcal{S}_n(t), U_n(t), \alpha_n(t))$, where $\mathcal{S}_n(t)$, $U_n(t)$, and $\alpha_n(t) \in \{c, r\}$ represent its strategy, utility, and personality, respectively. When implementing the utility-based learning model, during the decision period, each player only needs to evaluate its utility and capture the historical state and then choose a strategy. This model can be illustrated using the diagram shown in Fig. 2. Note that each player does not need to know the strategies adopted by the opponents, and in fact, it is not even aware of other players.

Considering its previous personality $\alpha_n(t-1)$ and action $\mathcal{S}_n(\mathrm{t}-1)$ at time t, player n will first determine its mixed strategy as

$$\mathbf{Q}_n = \left(q_1, q_2, \ldots, q_{|\mathbb{S}_n|}\right) \in \Delta(\mathbb{S}_n). \tag{10}$$

In (10), for each decision epoch t, $\Delta(\mathbb{S}_n)$ denotes the set of probability distribution over the strategy space $\mathbb{S}_n$, and q_i is the probability of choosing the strategy whose index in $\mathbb{S}_n$ is i, i.e.,

$$q_i \geq 0,\ \forall\, i \in \{1, 2, \ldots, |\mathbb{S}_n|\}, \quad \sum_{i=0}^{|\mathbb{S}_n|} q_i = 1.$$

Hence, the mixed strategy is applied to depict the dynamics of each player during the learning process.

Particularly, the detailed formulas for calculating the mixed strategy are given as follows. If $\alpha_n(t-1) = r$

$$\mathbf{Q}_n\left(i(\mathcal{F}_n)\right) = \frac{1}{|\mathbb{S}_n|},\ \forall\, \mathcal{F}_n \in \mathbb{S}_n \tag{11}$$

where $i(\mathcal{F}_n)$ is the index of strategy $\mathcal{F}_n$ in $\mathbb{S}_n$, and $\mathbf{Q}_n(i(\mathcal{F}_n))$ represents the $i(\mathcal{F}_n)$th entry in vector $\mathbf{Q}_n$. If $\alpha_n(t-1) = c$

$$\mathbf{Q}_n\left(i(\mathcal{F}_n)\right) = \begin{cases} \frac{\varepsilon^w}{|\mathbb{S}_n|-1} & \forall\, \mathcal{F}_n \in \mathbb{S}_n, \mathcal{F}_n \neq \mathcal{S}_n(t-1) \\ 1-\varepsilon^w, & \text{otherwise} \end{cases} \tag{12}$$

where ε is a constant belonging to [0, 1], and w is a constant larger than N. From (11) and (12), we note that the dynamics of the conservative player and those of the radical player are different. To be specific, the conservative will occasionally adopt new strategies with a small probability, but new strategies will be frequently employed by the radical.

After that, player n will choose an action $\mathcal{S}_n(t)$ according to the mixed strategy $\mathbf{Q}_n$, calculate its utility $U_n(t)$ by measuring the interference and then update the personality with Algorithm 1.

Algorithm 1 Personality updating algorithm

1: **if** $\alpha_n(t-1) = c$ **then**
2:   **if** $(\mathcal{S}_n(t) = \mathcal{S}_n(t-1))$ and $(U_n(t) = U_n(t-1))$ **then**
3:     **Set** $\alpha_n(t)$ to c
4:   **else**
5:     **Go** to Line 10.
6:   **end if**
7: **else**
8:   **Go** to Line 10.
9: **end if**
10: **Set** $\alpha_n(t)$ to c and r with the probability $\rho_c = \varepsilon^{1-(U_n(t)/F_n)^\beta}$ and $\rho_r = 1 - \rho_c$, respectively.

In Algorithm 1, F_n and β are constants adopted to normalize the utility, i.e., $\widehat{U}_n = (U_n(t)/F_n)^\beta \in (0, 1),\ \forall\, t$. Here, we will show how to choose the factor F_n and defer the study of the effect of β to the following section. Each FAP n can obtain the maximum achievable rate on each subchannel when there is no cotier interference, i.e.,

$$R^k_{n,\max} = B\log_2\left(1+\frac{p^k_n g^k_{n,n}}{B_0 N_0}\right)\ \forall\, k \in \{1, 2, \ldots, K\}. \tag{13}$$

Therefore, F_n can be set as

$$F_n = \max\left\{\sum_{l\in\mathcal{S}_n} R^l_{n,\max} | \mathcal{S}_n \subseteq \mathbb{S}_n\right\} \tag{14}$$

which is always larger than $U_n(t),\ \forall\, t$.

Making use of the described learning model, we can develop a DSA algorithm, which is termed as UDSA and depicted in Algorithm 2. In this algorithm, players can sequentially update their strategies. Similar to [36], the stop criterion of this algorithm can be one of the following: 1) The preset maximum iteration number T is reached; or 2) for each player n, the variation of its utility during a period is trivial.

Algorithm 2 UDSA

Set iteration count $t = 0$, personality $\alpha_n(t) = r$, and strategy count $\mathbf{C}_n = (\mathbf{0})_{1\times|\mathbb{S}_n|},\ \forall\, n \in \mathcal{N}$. Each player n randomly chooses a strategy $\mathcal{S}_n(t)$, and receives a utility $U_n(t)$.
repeat
  Set $t = t + 1$
  for $n = 1$ to N users **do**
    update state profile $\mathbb{L}_n(t)$:
    if $\alpha_n(t-1) = r$ **then**
      Calculate $\mathbf{Q}_n$ with (11).
    else
      Calculate $\mathbf{Q}_n$ with (12).
    end if

Choose a strategy $\mathcal{S}_n(t)$, measure the utility $U_n(t)$, and update its personality $\alpha_n(t)$.
Update strategies count $\mathbf{C}_n$:
if $\alpha_n(t) = c$ **then**
Update $\mathbf{C}_n$ with (15).
end if
end for
until the stop criterion is satisfied.
Each player decides its strategy according to (16).

At the beginning of UDSA, the related parameters and players' states should be initialized, where $(\mathbf{0})_{1\times M}$ represents an M-dimension null vector. After that, the algorithm goes into a loop. At each iteration t, player n will first update its state profile $\mathbb{L}_n(t) = (\mathcal{S}_n(t), U_n(t), \alpha_n(t))$ with the devised utility-based learning model. Then, it will update the strategy count $\mathbf{C}_n$ according to its current personality. If $\alpha_n(t) = c$

$$\mathbf{C}_n\left(i\left(\mathcal{S}_n(t)\right)\right) = \mathbf{C}_n\left(i\left(\mathcal{S}_n(t)\right)\right) + 1 \tag{15}$$

where $i(\mathcal{S}_n(t))$ is the index of $\mathcal{S}_n(t)$ in $\mathbb{S}_n$. Intuitively, the given rule means that each player will record the strategy that makes its personality c. When the loop is exited, individual players will make their final decisions as follows:

$$\mathcal{S}_n^D = \underset{\mathcal{S}_n}{\arg\max}\, \mathbf{C}_n(i(\mathcal{S}_n))\ \forall\, \mathcal{S}_n \in \mathbb{S}_n,\ \forall\, n \in \mathcal{N}. \tag{16}$$

From (16), we find that the strategy recorded most frequently will be eventually adopted.

It is seen that our proposed Algorithm 2 is simple and completely distributed. In particular, when each player updates its strategy, it does not require any prior information of other players, e.g., the CSI between different femtocells and the utility functions of its competitors. Next, we will analyze the complexity of this algorithm.

Recalling the proposed algorithm UDSA, which can be implemented in parallel, we note that each player only needs to make its own decision, and meanwhile, only basic arithmetic operations and random number generation are involved in each iteration step. Therefore, the complexity of this algorithm is depending on both the stop criterion of the loop and the sizes of players' strategy spaces. Particularly, for the two different stop criteria previously described, the complexities are $O(T+L)$ and $O(E+L)$, respectively, where T is the preset maximum iteration number, $L = \max\{|\mathbb{S}_1|, |\mathbb{S}_2|, \ldots, |\mathbb{S}_N|\}$, and E is the convergence rate of the algorithm. In addition, it should be noted that the convergence rate E is related to the algorithm parameters, which will be further discussed in the following section.

C. Performance Analysis of UDSA

Here, the performance of the developed algorithm will be analyzed. First, we give the following theorem, which indicates that UDSA can asymptotically converge to the solution maximizing the aggregate normalized utility under the given condition.

Theorem 1: For NRMG, let $(\mathcal{S}_n^O)_{n\in\mathcal{N}} \in (\mathbb{S}_n)_{n\in\mathcal{N}}$ denote the profile of strategies satisfying

$$\left(\mathcal{S}_n^O\right)_{n\in\mathcal{N}} = \underset{(\mathcal{S}_n)_{n\in\mathcal{N}}}{\arg}\ \max \hat{U}\left((\mathcal{S}_n)_{n\in\mathcal{N}}\right) \tag{17}$$

where

$$\begin{aligned}\hat{U}\left((\mathcal{S}_n)_{n\in\mathcal{N}}\right) &= \sum_{n=1}^{N} \hat{U}_n\left((\mathcal{S}_n)_{n\in\mathcal{N}}\right)\\ &= \sum_{n=1}^{N}\left(\frac{U_n\left((\mathcal{S}_n)_{n\in\mathcal{N}}\right)}{F_n}\right)^{\beta}.\end{aligned} \tag{18}$$

When $(\mathcal{S}_n^O)_{n\in\mathcal{N}}$ is unique and ε is sufficiently small, i.e., $\varepsilon \to 0$, UDSA asymptotically converges to $(\mathcal{S}_n^O)_{n\in\mathcal{N}}$, i.e.,

$$\Pr\left(\lim_{T\to\infty,\varepsilon\to 0}\left(\mathcal{S}_n^D\right)_{n\in\mathbb{N}} = \left(\mathcal{S}_n^O\right)_{n\in\mathcal{N}}\right) = 1 \tag{19}$$

where T is the number of iterations.

Proof: The proof is given in the Appendix. ■

In general, for distributed algorithms, the effectiveness of the achieved solutions is generally evaluated using Pareto optimality or known as Pareto efficiency, which is introduced in the following definition.

Definition 3—Pareto Optimality (Pareto Efficiency): For a situation, a profile

$$\mathcal{S}^{\mathrm{PO}} = \left(\mathcal{S}_1^{\mathrm{PO}}, \mathcal{S}_2^{\mathrm{PO}}, \ldots, \mathcal{S}_N^{\mathrm{PO}}\right)$$

in the strategy space is Pareto optimal (Pareto efficient), if and only if there exists no other set of strategies for which at least one player can improve its own welfare without reducing those of the other users.

We refer to the condition that $\varepsilon \to 0$ and $(\mathcal{S}_n^O)_{n\in\mathbf{N}}$ is unique as the ideal condition. Then, combining the discussions in Theorem 1 and Definition 3, we have the following corollary.

Corollary 1: For NRMG, $\Gamma(\mathcal{N}, (\mathbb{S}_n)_{n\in\mathcal{N}}, (U_n)_{n\in\mathcal{N}})$, the solution achieved by UDSA, i.e., $(\mathcal{S}_n^D)_{n\in\mathcal{N}}$, is Pareto optimal when the ideal condition is met.

Proof: This corollary can be proved via the *reductio ad absurdum* approach. We assume that $(\mathcal{S}_n^D)_{n\in\mathcal{N}}$ is not Pareto optimal. Mathematically speaking, $\exists(\tilde{\mathcal{S}}_n)_{n\in\mathcal{N}} \in \mathbb{S}_n$

$$\hat{U}_n\left((\tilde{\mathcal{S}}_n)_{n\in\mathcal{N}}\right) \geq \hat{U}_n\left(\left(\mathcal{S}_n^D\right)_{n\in\mathcal{N}}\right)\ \forall\, n \in \mathcal{N} \tag{20}$$

and meanwhile

$$\hat{U}_m\left((\tilde{\mathcal{S}}_n)_{n\in\mathcal{N}}\right) > \hat{U}_m\left(\left(\mathcal{S}_n^D\right)_{n\in\mathcal{N}}\right),\ \exists\, m \in \mathcal{N}. \tag{21}$$

Then, we will get

$$\sum_{n=1}^{N}\left(\frac{U_n\left((\tilde{\mathcal{S}}_n)_{n\in\mathcal{N}}\right)}{F_n}\right)^{\beta} > \sum_{n=1}^{N}\left(\frac{U_n\left(\left(\mathcal{S}_n^D\right)_{n\in\mathcal{N}}\right)}{F_n}\right)^{\beta} \tag{22}$$

which contradicts Theorem 1.

Therefore, $(\mathcal{S}_n^D)_{n\in\mathcal{N}}$ is Pareto optimal. Now, we complete the proof. ■

According to Corollary 1, when $(\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}}$ is achieved, the overall capacity

$$R = \sum_{n=1}^{N} R_n = \sum_{n=1}^{N} U_n \tag{23}$$

cannot be further improved without reducing the capacity of any one femtocell. In other words, this solution will result in win-win outcomes for both the femtocell owners and the operator. We also note that there is no requirement that $(\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}}$ is the NE for NRMG. Otherwise stated, it can happen that this efficient point may be ignored by the distributed scheme, whose aim is to reach an NE for the game.

As the end of this section, we shift our focus to study the effects of parameters β and ε, which are adopted in the proposed algorithm. In fact, when the parameter profile $\{F_1, F_2, \ldots, F_N\}$ is selected, the concerned Pareto-optimal solution $(\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}}$ can also be achieved by solving the following integer programming:

$$\begin{aligned}
\underset{(\mathcal{S}_n)_{n\in\mathcal{N}}}{\arg\max} \quad & \widehat{U} = \sum_{n=1}^{N} \widehat{U}_n \\
& = \sum_{n=1}^{N} \left(\frac{R_n\left((\mathcal{S}_n)_{n\in\mathcal{N}}\right)}{F_n} \right)^{\beta} \\
\text{s.t.}: \quad & \mathcal{S}_n \subset \{1, 2, \ldots, K\} \\
& |\mathcal{S}_n| = K_n \;\forall\, n \in \{1, 2, \ldots, N\}.
\end{aligned} \tag{24}$$

For notational simplicity, we denote by $\widehat{U}_{\mathrm{PO}}$ and R_{PO} the corresponding aggregate normalized utility and capacity, respectively, i.e.,

$$\widehat{U}_{\mathrm{PO}} = \sum_{n=1}^{N} \widehat{U}_n \left((\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}} \right) \tag{25}$$

$$R_{\mathrm{PO}} = \sum_{n=1}^{N} R_n \left((\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}} \right). \tag{26}$$

It can be seen from the problem shown in (24) that the value of β will characterize both the utility $\widehat{U}_{\mathrm{PO}}$ and the capacity R_{PO}. In specific, $\widehat{U}_{\mathrm{PO}}$ is monotonically decreasing with respect to β. Nevertheless, the relationship between β and R_{PO} is not obvious and intractable for mathematical analysis.

On the other hand, both the values of β and ε will affect the convergence of UDSA. Smaller ε will lead to a slower convergence speed, but the algorithm is more likely to converge to the Pareto-optimal solution $(\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}}$. In addition, the value of β will also affect the performance of UDSA. As shown in line 10 of Algorithm 1, smaller β will make players set their personality to c more frequently. This updating rule eventually implies that each player n may act "irrationally" and that the algorithm will converge faster. The given comments will be illustrated with simulation results in Section V.

Remark 1: According to Theorem 1, a prerequisite to achieve $(\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}}$ with UDSA is $\varepsilon \to 0$. In fact, to balance the algorithm performance and its convergence rate, ε cannot be set too small. However, our strategy can also bring outstanding performance in different interference environments, which will be shown through simulation, as shown in the following section.

TABLE I
SIMULATION PARAMETERS

Parameter	**value**
Number of subchannels, K	5
Region radius, r	100 m
Maximum distance between each FAP-FU pair, D	10 m
Bandwidth per subchannel, B_0	100 kHz
Subchannel requirement, K_n, $\forall n$	3
FAP power limit, $p_{n,max}$, $\forall n$	20dBm
Standard deviation of ψ, σ_ψ	4 dB
Wall penetration loss, L_{wall}	5 dB
Noise figure, φ_{FU}	7 dB
AWGN power density, N_0	-174 dBm/Hz

V. RESULTS AND ANALYSIS

A. Simulation Scenario

We consider a circular region of radius r m, where N femtocells are randomly deployed. In each femtocell, the distance between a FAP-FU pair is a uniform random variable between 0 and D m. Furthermore, for each FAP-FU pair, the channel gains are independent on different subchannels. During the transmission period, we employed the uniform distribution mechanism to determine the power set, i.e.,

$$p_n^l = \frac{p_{n,\max}}{K_n} \;\forall\, l \in \mathcal{S}_n \;\forall\, \mathcal{S}_n \in \mathbb{S}_n \;\forall\, n \in \mathcal{N}.$$

Unless specified otherwise, the simulation parameters are shown in Table I [28], [32], and each individual simulation result is obtained by averaging over 10 000 independent runs.

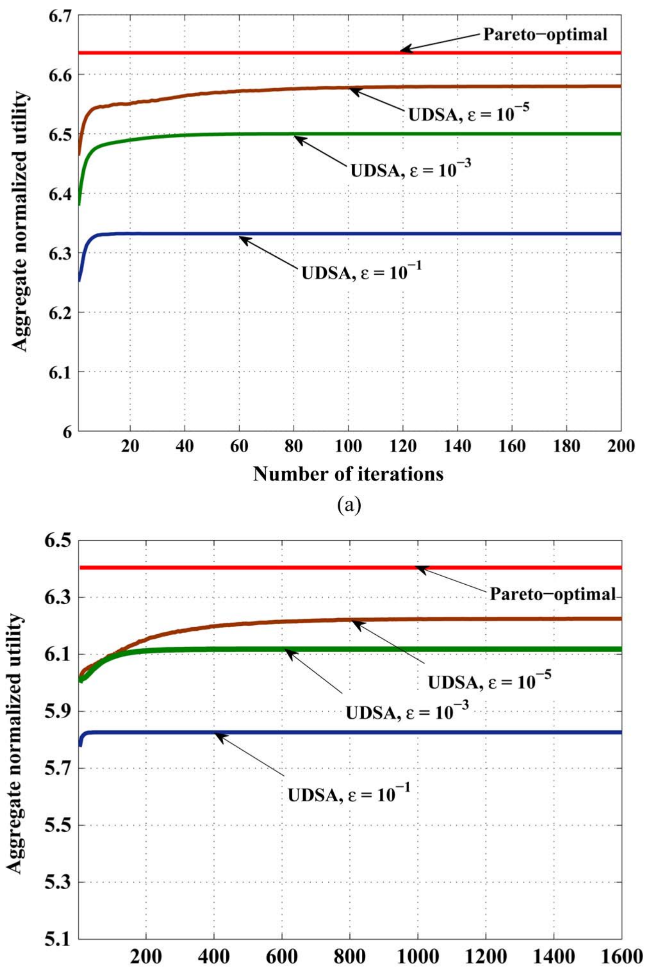
B. Convergency

To verify the validity of our analysis and evaluate the convergence of the proposed algotirhm, we first calculate the values of $\widehat{U}_{\mathrm{PO}}$ and R_{PO} when β is set to different values. Then, we will explore the difference between $\widehat{U}_{\mathrm{PO}}$ and $\widehat{U}_D$, which is the normalized utility achieved by UDSA. Note that there is no efficient algorithm to solve the integer programming shown in (24), and hence, the method of exhaustion is used here, which needs to compare all the $\prod_{n\in\mathcal{N}} |\mathbb{S}_n|$ utilities to obtain the optimal solution. To find $(\mathcal{S}_n^{\mathrm{PO}})_{n\in\mathcal{N}}$ within an acceptable period of time, we consider a small-scale scenario as an example, where seven femtocells are deployed in a random fashion, and for each FU n, $K_n = 1$.

Table II demonstrates the value of $\widehat{U}_{\mathrm{PO}}$ and R_{PO} as β changes from 0.1 to 1. It is obvious that the larger β leads to the smaller $\widehat{U}_{\mathrm{PO}}$. However, the variation pattern of R_{PO} is difficult to characterize, which is consistent with our previous comments in Section IV-C. For instance, $R_{\mathrm{PO}}(\beta = 0.1) < R_{\mathrm{PO}}(\beta = 0.3)$, $R_{\mathrm{PO}}(\beta = 0.3) > R_{\mathrm{PO}}(\beta = 0.5)$, but $R_{\mathrm{PO}}(\beta = 0.5) < R_{\mathrm{PO}}(\beta = 0.7)$. Then, we illustrate the convergence of UDSA in Fig. 3, where β is set to 0.1 and 0.2, respectively.

TABLE II
VALUE OF $\widehat{U}_{\text{PO}}$ AND R_{PO} VERSUS β

β	$\widehat{U}_{PO}$	R_{PO} (Mbps)	β	$\widehat{U}_{PO}$	R_{PO} (Mbps)
0.1	6.64	15.76	0.2	6.40	15.93
0.3	6.07	15.96	0.4	5.88	16.02
0.5	5.60	15.46	0.6	5.32	15.83
0.7	5.30	15.79	0.8	5.03	15.88
0.9	4.93	15.54	1.0	4.90	16.01



(a)

(b)

Fig. 3. Convergence of UDSA with respect to ε for (a) $\beta = 0.1$ and (b) $\beta = 0.2$.

It is seen that when β and ε are given, the development of our algorithm follows a monotonically increasing path before stabilizing. Moreover, smaller ε results in longer convergence time for given β, but the stable $\widehat{U}_D$ is closer to $\widehat{U}_{\text{PO}}$, which is illustrated by the red solid curve. For example, in Fig. 3(a), when $\varepsilon = 10^{-1}$ and $\varepsilon = 10^{-5}$, the convergence time is about 10 and 100 iterations, respectively. In addition, when ε is set to 10^{-5}, the achieved $\widehat{U}_D$ is approximately 3.95% higher than that of $\varepsilon = 10^{-1}$. Meanwhile, we note that, in the case that $\beta = 0.1$ and $\varepsilon = 10^{-5}$, there still is a small difference between $\widehat{U}_D$ and $\widehat{U}_{\text{PO}}$, and specifically, the relative gap is around 0.81%.

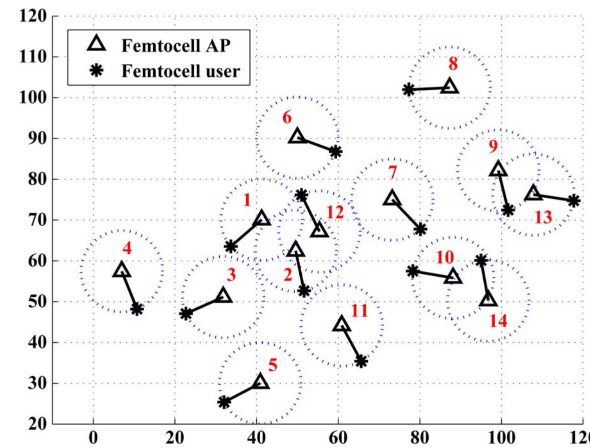


Fig. 4. Simulation snapshot where 14 OFDMA femtocells are deployed.

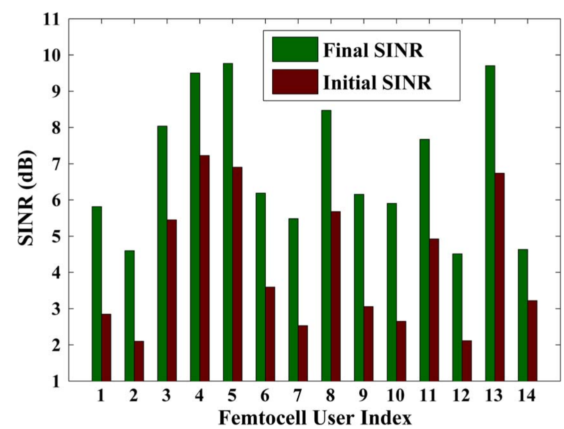


Fig. 5. Comparison of average SINR in decibels for FUs. At each FU n, the SINR in decibels is $\gamma_n = \sum_{l \in \mathcal{S}_n} 10 \log_{10}(\gamma_n^l)$.

Theoretically, this difference happens for two reasons: First, ε is not small enough; second, there is no guarantee that the Pareto-optimal solution is always unique in each round of simulation. We would also note that the convergence speed is reduced with the increase of β when ε is given. For instance, when $\varepsilon = 10^{-5}$ and $\beta = 0.1$, the convergence rate is nearly ten times faster than that in the case that $\varepsilon = 10^{-5}$ and $\beta = 0.2$.

C. Performance Evaluation

To demonstrate the interference mitigation capability of UDSA, we first consider a simulation snapshot shown in Fig. 4. Without confusion, we label the FAP and FU with the corresponding femtocell's index. Each FAP randomly choose subchannels during the initialization of the simulation, and then, UDSA is implemented. In this simulation, we set $K_n = 1\ \forall n \in \mathcal{N}$, $\beta = 0.1$, and $\varepsilon = 0.01$. The initial and final SINRs in decibels at FUs are presented in Fig. 5. It is noticed that there is an overall improvement of SINR, which means that every FAP can autonomously enhance the FU's SINR with the proposed interference mitigation scheme. Particularly, the average SINR increases from 4.22 to 6.89 dB. Furthermore, we find that the received SINR in femtocells 2 and 12 is much smaller

TABLE III
INFORMATION REQUIREMENT OF DSA SCHEMES

Schemes	DSA-[27]	DSA-[28]	UDSA
CSI between different cells	√	√	×
Other players' strategies	√	√	×
Other players' transmit power	√	√	×
Incoming interference	×	×	√

than the average value. That is because these two femtocells are surrounded by more interference neighbors. In addition to SINR, the sum capacity of femtocells $R = \sum_{n=1}^{N} R_n$ is also recorded, and the data show that R has been improved by around 20.9%, which is from 3.73 to 4.51 Mb/s.

To further evaluate our strategy, two DSA schemes are compared, which have been proposed in [27, Sec. IV] and [28], respectively. For notional simplicity, we denote them as DSA-[27] and DSA-[28]. On one hand, we compare them in terms of the required information, which is shown in Table III. Particularly, using DSA-[27] and DSA-[28], each FAP has to know the CSI from other FAPs to its own FU and that from itself to the FUs associating to other FAPs. In addition, for each individual FAP, it is necessary to know the strategies adopted by the other FAPs to choose its own best response action in the each iteration of the developed algorithm. However, one FAP only needs to measure the incoming interference and calculate its own utility when implementing UDSA. As a result, no information exchange is required among different femtocells.

On the other hand, we investigate the performance of UDSA in terms of the average SINR per subchannel in decibels, i.e.,

$$\bar{\gamma} = \frac{1}{N}\sum_{n=1}^{N}\frac{\sum_{l\in\mathcal{S}_n} 10\log_{10}\left(\gamma_n^l\right)}{K_n} \tag{27}$$

and of the overall capacity R, which are shown in Fig. 6(a) and (b), respectively. In this simulation, femtocells are randomly deployed, and moreover, K_n, β, and ε are set to 3, 0.1, and 0.01, respectively. In addition to DSA-[27] and DSA-[28], there is another method that should be compared, with which each FAP n will randomly utilize subchannels. We term this strategy as DSA-Random and regard its performance as the baseline.

As shown in Fig. 6(a), the average SINR is decreasing when there are more active femtocells. This is for the reason that more femtocells will result in higher interference. Furthermore, we note that, although there is no information exchange among the FAPs, the proposed algorithm can bring the highest SINR. In other words, the interference mitigation capability of our strategy is better than that of existing strategies. Compared with DSA-Random, DSA-[27], and DSA-[28], UDSA has around 45.7%, 16.8%, and 9.8% higher average SINR, respectively.

From the perspective of overall capacity, we have compared UDSA with the available strategies in different interference environments, i.e., the number of communicating femtocells N is set to different values. As demonstrated in Fig. 6(b), our scheme always brings higher sum capacity, which is accordant with the result shown in Fig. 6(a). For instance, when $N = 50$, UDSA provides approximately 11.0% more capacity than that

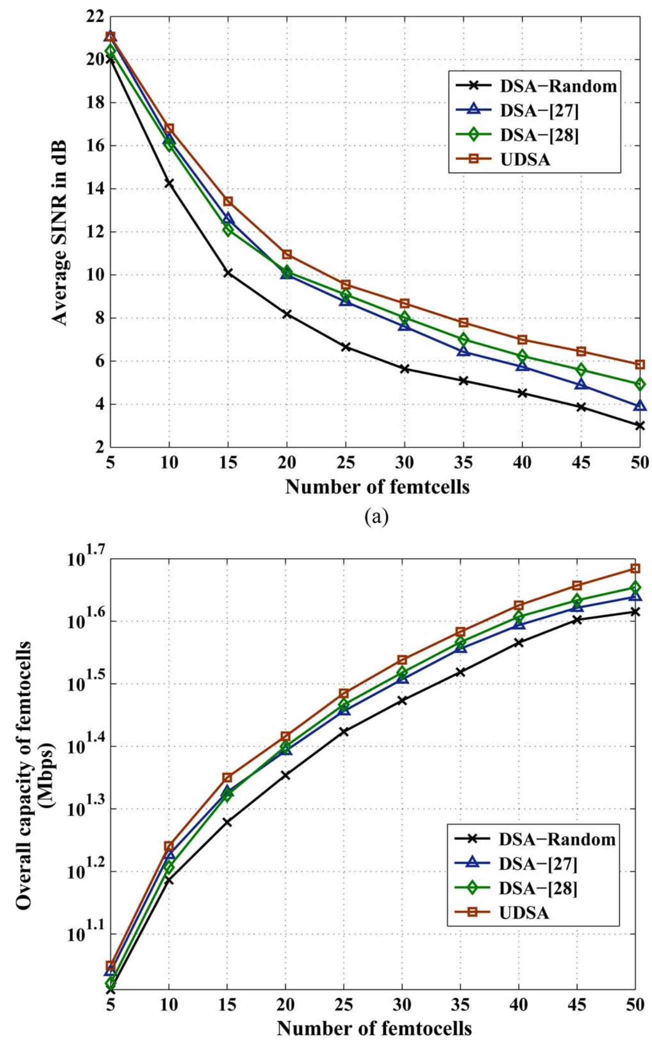


Fig. 6. Performance comparison in terms of (a) the average SINR $\bar{\gamma}$ and (b) the overall capacity R.

of DSA-[27] and 7.2% more capacity than that of DSA-[28]. The reason for this improvement is that the purpose of each player in this paper is to maximize its capacity. However, in both DSA-[27] and DSA-[28], the relationship between utility and capacity is not obvious. Meanwhile, from the simulation results, we note that the improvement of the overall capacity is gradually slowed down when the density of femtocells becomes high. The similar observation has also been made in previous work [37], where it is referred to as the fundamental throughput scaling limit.

D. More Realistic Scenario

Here, we demonstrate the performance of our algorithm in more realistic LTE femtocells, where the system bandwidth is 3 MHz (i.e., $K = 15$) and the bandwidth of each RB is 180 kHz. Throughout this section, for every user $n \in \mathcal{N}$, we consider $K_n = K/5 = 3$, i.e., the size of strategy space $|\mathbb{S}_n|$ is 455. In addition, the simulation result is obtained by averaging over 1000 independent runs.

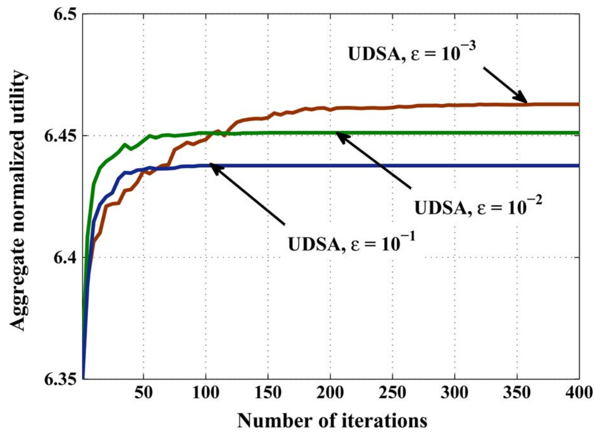


Fig. 7. Convergence of UDSA with respect to ε for $\beta = 0.1$.

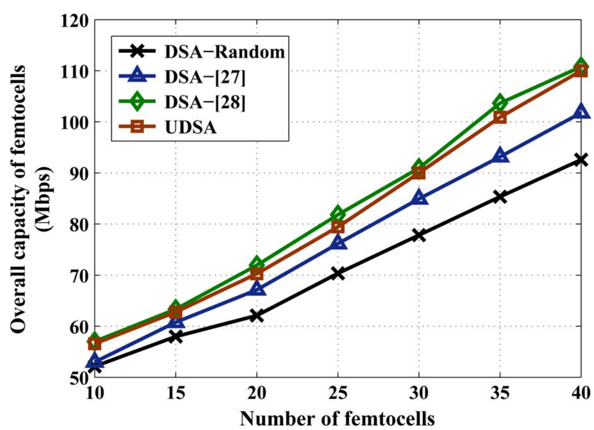


Fig. 8. Performance comparison in terms of overall capacity R for LTE femtocells, where the total bandwidth is 3 MHz.

To compare with the results shown in Fig. 3, here, we also set $N = 7$ and $\beta = 0.1$ and then illustrate the convergence of our algorithm in Fig. 7. We note that, different from Fig. 3, the Pareto-optimal utility $\widehat{U}_{\rm PO}$ is not shown here. The reason is that $\widehat{U}_{\rm PO}$ can only be obtained after comparing all $\prod_{n\in\mathcal{N}} |\mathbb{S}_n| = 4.037 \times 10^{18}$ strategy profiles, but such a huge amount of enumeration cannot be performed within a reasonable time. From the simulation results, it is seen that a tradeoff between efficiency and convergence rate can also be made by adjusting parameter ε. In addition, compared with the results presented in Fig. 3, we note that a larger size of strategy for each player will make the convergence of our algorithm slower. This is due to the fact that the subcahnnel allocation is essentially a combinatorial problem. The similar observation can also be made in previous distributed algorithms [27], [28], [36].

Finally, we compare the performance of our scheme with that of the three available schemes, i.e., DSA-Random, DSA-[27], and DSA-[28]. In particular, as the number of femtocells N increases, the performance in terms of overall capacity R is demonstrated in Fig. 8, where β and ε are set to 0.1 and 0.01, respectively. From the simulation results, we can see that our devised method has roughly the same performance as DSA-[28] but without information exchange among different femtocells. On the other hand, compared with DSA-[27], even better performance can be obtained by our scheme. For instance, when $N = 40$, UDSA yields a performance advantage around 8.2% relative to the scheme DSA-[27], i.e., from 101.62 to 110.04 Mb/s. It should be noted that the reasons for this improvement are similar to those previously described, and moreover, the simulation results presented here also show the advantage of our approach.

VI. Conclusion

In this paper, we have addressed the issue of DSA for interference mitigation in OFDMA femtocells and proposed a UDSA algorithm. The developed algorithm is appropriate for the networks that are organized in an ad hoc fashion, since there is no information interaction among the autonomous agents and moreover, the Pareto-optimal resource allocation can be achieved under the given condition. Simulation results verify the validity of our analysis and demonstrate the effectiveness of our proposed scheme. Compared with the available strategies requiring information exchange, our approach achieves comparable or even better performance in different interference environments. Here, we just concern ourselves with cotier interference and defer the study of interference mitigation in two-tier femtocell networks in our future work.

Appendix
Proof of Theorem 1

The learning model adopted in Algorithm 2 introduces a Markov process over the finite state space $\mathcal{Z} = \prod_{n\in\mathcal{N}}(\mathbb{S}_n \times \hat{\mathcal{U}}_n \times \mathcal{D}_n)$, where $\hat{\mathcal{U}}_n$ is the finite range of

$$\hat{U}_n = \left(\frac{U_n\left((\mathcal{S}_n)_{n\in\mathcal{N}}\right)}{F_n}\right)^{\beta} \tag{28}$$

over all $(\mathcal{S}_n)_{n\in\mathcal{N}} \in (\mathbb{S}_n)_{n\in\mathcal{N}}$, and $\mathcal{D}_n = \{c, r\}$ is the set of personality for each player n. Accordingly, for any scalar $\varepsilon > 0$, such a Markov process can be regarded as a "perturbed" process, which we denote by $\mathrm{MP}^{\varepsilon}$. Before giving the proof in detail, we start by introducing some necessary definitions.

Definition 4—Interdependence: An n-person game $\Gamma(\mathcal{N}, (\mathbb{S}_n)_{n\in\mathcal{N}}, (U_n)_{n\in\mathcal{N}})$ is interdependent if, for every strategy profile $(\mathcal{S}_n)_{n\in\mathcal{N}} \in (\mathbb{S}_n)_{n\in\mathcal{N}}$ and every nonempty proper subset $\mathcal{M}$ of players in $\mathcal{N}$, i.e., $\forall\, \mathcal{M} \subset \mathcal{N}$, there exists a player $g \notin \mathcal{M}$ and a choice of strategies $(\mathcal{S}'_m)_{m\in\mathcal{M}} \in (\mathbb{S}_m)_{m\in\mathcal{M}}$ such that

$$U_g\left((\mathcal{S}'_n)_{m\in\mathcal{M}}, (\mathcal{S})_{n\in\{\mathcal{N}/\mathcal{M}\}}\right) \neq U_g\left((\mathcal{S}_m)_{m\in\mathcal{M}}, (\mathcal{S})_{n\in\{\mathcal{N}/\mathcal{M}\}}\right). \tag{29}$$

In other words, (29) means that, given any strategy profile $(\mathcal{S}_n)_{n\in\mathcal{N}} \in (\mathbb{S}_n)_{n\in\mathcal{N}}$, every nonempty proper subset $\mathcal{M} \subset \mathcal{N}$ can cause a welfare change for some player in $\{\mathcal{N}/\mathcal{M}\}$ by suitably changing the actions of its elements, i.e., players. Intuitively, this definition indicates that, for an interdependent game, it is impossible to divide the players into two disjoint subsets that do not mutually interact with each other [11].

Definition 5—Stochastically Stable States: For a perturbed Markov process, the support of stationary distribution is referred to as the set of stochastically stable states. Specifically, a state $\mathcal{T} \in \mathcal{Z}$ is stochastically stable if and only if $\lim_{\varepsilon\to 0} \pi(\mathcal{T}, \varepsilon) > 0$, where $\pi(\mathcal{T}, \varepsilon)$ is the stationary distribution of the process.

We divide the proof of Theorem 1 into three steps, **S1–S3**, which are formally elaborated as follows.

Step S1

First, we will reconstruct a new game $\hat{\mathcal{G}} = \Gamma(\mathcal{N}, (\mathbb{S}_n)_{n\in\mathcal{N}}, (\hat{U}_n)_{n\in\mathcal{N}})$, where $\mathcal{N}$ and $(\mathbb{S}_n)_{n\in\mathcal{N}}$ have the same definitions given in Definition 1, and $\hat{U}_n$ is the normalized utility (welfare) shown in (28).

Proposition 1: $\hat{\mathcal{G}}$ is an interdependent game.

Proof: Given the femtocell scenario, according to (6) and (28), the normalized utility of each player is based on its capacity. We consider a situation that a single player m who can change its strategy, i.e., utilizing another set of subchannels, and meanwhile, other players stay the same. Let us suppose that the current strategy profile is $(\mathcal{S}_n)_{n\in\mathcal{N}}$, and then, the according situation can be divided into three disjoint cases: 1) $\forall n \in \{\mathcal{N}/\{m\}\}$, $\mathcal{S}_m \bigcap \mathcal{S}_n = \emptyset$; 2) $\forall n \in \{\mathcal{N}/\{m\}\}$, $\mathcal{S}_m \cap \mathcal{S}_n \neq \emptyset$; 3) $\exists n \in \{\mathcal{N}/\{m\}\}$, $\mathcal{S}_m \bigcap \mathcal{S}_n = \emptyset$, and meanwhile, $\exists q \in \{\mathcal{N}/\{m\}\}$, $\mathcal{S}_m \cap \mathcal{S}_q \neq \emptyset$. Here, $\emptyset$ denotes the empty set.

In the first case, for every player $g \in \{\mathcal{N}/\{m\}\}$, if player m changes its strategy to $\mathcal{S}'_m$, which satisfies $\mathcal{S}'_m \bigcap \mathcal{S}_g \neq \emptyset$, player g will suffer higher interference and achieve lower capacity. Then, its welfare (utility) will be reduced. In the second case, since $0 < K_n < K$, $\forall n \in N$, for each player $g \in \{\mathcal{N}/\{m\}\}$, we have $1 \leq |\mathcal{S}_m \cap \mathcal{S}_g| \leq \min\{K_m, K_g\} < K$. Therefore, m can change its strategy to satisfy $|\mathcal{S}'_m \cap \mathcal{S}_g| < |\mathcal{S}_m \cap \mathcal{S}_g|$ (or $|\mathcal{S}'_m \cap \mathcal{S}_g| > |\mathcal{S}_m \cap \mathcal{S}_g|$), which implies that player g will suffer lower (or higher) interference, and its utility will be improved (or reduced). For instance, if $|S_m \cap S_g| = K_m$ or $|S_m \cap S_g| = K_g$, we can obtain $|\mathcal{S}'_m \cap \mathcal{S}_g| < |\mathcal{S}_m \cap \mathcal{S}_g|$ by properly changing the strategy of player m from $\mathcal{S}_m$ to $\mathcal{S}'_m$. On the other hand, if $|S_m \cap S_g| < \min\{K_m, K_g\}$, we can obtain $|\mathcal{S}'_m \cap \mathcal{S}_g| = \min\{K_m, K_g\} > |\mathcal{S}_m \cap \mathcal{S}_g|$ by properly changing the strategy of player m from $\mathcal{S}_m$ to $\mathcal{S}'_m$. In the third case, similar to the discussions made in the two given cases, it is easy to prove that the welfare of every player $g \in \{\mathcal{N}/\{m\}\}$ can be effected if player m properly changes its strategy.

The given analysis shows that in all three cases, every player $m \in \mathcal{N}$ can change each opponent's utility by properly adopting another strategy. Therefore, for every nonempty proper subset $\mathcal{M} \subset \mathcal{N}$, even if only one player $m \in \mathcal{M}$ properly changes its strategy, we can find a player $g \in \mathcal{N}/\mathcal{M}$ whose utility can be effected.

Hence, the game $\hat{\mathcal{G}}$ is interdependent. ■

Step S2

Proposition 2: For the introduced perturbed Markov process $\mathrm{MP}^{\varepsilon} = \mathcal{Z} = \prod_{n\in\mathcal{N}}(\mathbb{S}_n \times \hat{\mathcal{U}}_n \times \mathcal{D}_n)$, if and only if a state $\mathcal{T} = ((\mathcal{S}_n)_{n\in\mathcal{N}}, (\hat{u}_n)_{n\in\mathcal{N}}, (\alpha_n)_{n\in\mathcal{N}}) \in \mathcal{Z}$ is stochastically stable, then the strategy profile can maximize the social welfare, i.e.,

$$(\mathcal{S}_n)_{n\in\mathcal{N}} = \underset{(\mathcal{S}'_n)_{n\in\mathcal{N}}}{\arg} \max \sum_{n\in\mathcal{N}} \hat{U}_n\left((\mathcal{S}'_n)_{n\in\mathcal{N}}\right). \tag{30}$$

Moreover, in such a state, we have $\alpha_n = c$ and $\hat{u}_n = \hat{U}_n((\mathcal{S}_n)_{n\in\mathcal{N}})$, $\forall n \in \mathcal{N}$.

Proof: Based on the conclusion of Proposition 1 in this work and the proof of [11, Th. 1], Proposition 2 can be proved with the theory of resistance trees, and the proof is omitted here due to space limitations. ■

Step S3

If the social optimal solution is unique, there is just one stochastically stable state for the perturbed Markov process $\mathrm{MP}^{\varepsilon}$. According to Proposition 2, if the stochastically stable state $\mathcal{T} = ((\mathcal{S}_n)_{n\in\mathcal{N}}, (\hat{U}_n)_{n\in\mathcal{N}}, (\alpha_n)_{n\in\mathcal{N}}) \in \mathcal{Z}$ is unique, then we have

$$\begin{aligned}
&\lim_{\varepsilon\to 0} \pi(\mathcal{T}, \varepsilon) \\
&\quad = \lim_{\varepsilon\to 0} \Pr\left((\mathcal{S}_n)_{n\in\mathcal{N}}, (c)_{n\in\mathcal{N}}\right) = \lim_{\varepsilon\to 0} \prod_{n\in\mathcal{N}} \Pr\left(\mathcal{S}_n, (c)_{n\in\mathcal{N}}\right) \\
&\quad = \lim_{T\to\infty, \varepsilon\to 0} \prod_{n\in\mathcal{N}} \frac{t(\mathcal{S}_n, c)}{T} = 1
\end{aligned} \tag{31}$$

where $t(\mathcal{S}_n, c)$ is the frequency that the according state has occurred during period T. In Algorithm 2, each player will choose the most frequently recorded strategy that makes its personality c [as shown in (16)]. Therefore, the unique efficient strategy profile can be achieved with the proposed DSA algorithm UDSA.

The proof is complete. ■

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