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Versatile and robust chaos synchronization phenomena imposed by delayed shared feedback coupling

Michael Peil,[1] Laurent Larger,[1] and Ingo Fischer[2]
[1]*UMR CNRS FEMTO-ST 6174/Optics Department, Université de Franche-Comté, 16 Route de Gray, 25030 Besançon cedex, France*
[2]*School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom*

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In a joint experimental and modeling approach we demonstrate chaos synchronization imposed by a delayed shared feedback coupling between two nonlinear electro-optic oscillators. Robust identical synchronization is obtained for both symmetric and strongly asymmetric timing of the mutual coupling, offering great potential for applications such as chaos-based communications. We further demonstrate antisynchronization as well as generalized synchronization with vanishing linear correlation, by detuning the nonlinearity in one of the oscillators.

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The dynamics and synchronization properties of delay-coupled nonlinear oscillators are of importance in various fields of science—e.g., in brain dynamics [1–3], population dynamics [4,5], and coupled semiconductor lasers [6]. Not even two decades ago this paradigmatic system was first studied, since delayed coupling renders the system infinite dimensional, complicating the analysis. Since then, manifold influences of delayed coupling have been identified, comprising multistability of synchronized and desynchronized solutions [7], amplitude death [8], and delay-induced instabilities in conjunction with symmetry breaking. For the latter, it has been found that two coupled systems can synchronize, but exhibit a relative time lag, roughly corresponding to the coupling delay [6]. This has been overcome by introducing a relay element in the coupling path. By this, zero-lag synchronization has been demonstrated [9]. Analytic approaches underline the stability of such synchronization over large parameter regimes [10,11]. Also other laser configurations exhibiting zero-lag synchronization have been reported, comprising unidirectional coupling [12] or self-feedback [13,14]. In addition to its fundamental importance, chaos synchronization of delay-coupled systems is currently being discussed as a mechanism to realize new concepts for bidirectional chaos communication and key exchange [15,16]. It therefore can serve functional purposes.

In this paper we introduce a versatile system of mutually delay-coupled electro-optic oscillators, allowing for robust, multi-GHz-bandwidth chaos synchronization. By implementing a delayed shared feedback-coupling (DSFC) scheme we can demonstrate different types of synchronization comprising identical synchronization with time offset and antisynchronization and generalized synchronization with vanishing linear correlation.

The experimental setup is sketched in Fig. 1, showing the two mutually coupled electro-optic nonlinear delay oscillators, labeled as $i=1,2$. Following the principle of [17,18], in each oscillator a nonlinear transformation is performed by the two-wave interference modulation transfer function of an integrated-optics $LiNbO_3$ Mach-Zehnder modulator ($MZ_{1,2}$). Two circulators $C_{1,2}$ are connected to two fiber channels joined into the same 2×1 50/50 fiber coupler, the single output of which is cleaved and metallized to form a mirror. This configuration combines the two intensity dynamics from the MZ outputs, and it also performs the so-called DSFC for both oscillators. The DSFC signals are detected in both oscillators by broadband photodiodes $PD_{1,2}$, then amplified by broadband rf drivers, to finally serve as the modulating input for $MZ_{1,2}$, thus closing the oscillator loops. The linear electronic filtering actually ruling each oscillator differential process is mainly limited by the amplified photodiode bandwidth of 30 kHz to 12 GHz. Notice the important role of the mirror at the single end of the fiber coupler, which performs a balanced superposition of each oscillator dynamics, the latter superposition being then symmetrically redistributed to both oscillators. Delay is mainly determined by the fiber lengths connecting the different elements, which lengths can be chosen identical or asymmetric (e.g., inserting a fiber spool), in order to explore different timing of the mutual DSFC. Fine-tuning of the delays and feedback amplitudes are allowed by two variable-delay lines $VD_{1,2}$ and two variable attenuators $VA_{1,2}$. This coupling scheme is related to a recently proposed all-optical scheme [15], but it has been here adapted and generalized using electro-optic nonlinear delay oscillators. An advantage is that sensitivity against variation of optical phases can be avoided.

The light of each MZ output is observed via a broadband-amplified photodiode $PD'_{1,2}$ which is located at the free output of the 2×2 fiber coupler. $PD'_{1,2}$ are of the same model as $PD_{1,2}$. The dynamical states of both oscillators are analyzed

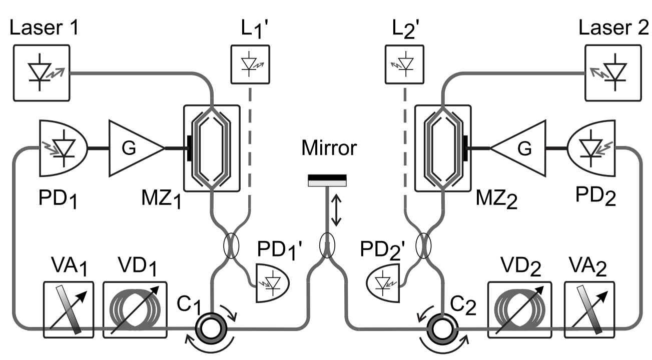

FIG. 1. Experimental setup of the electro-optic oscillators with delayed shared feedback coupling (DSFC).

simultaneously by connecting $PD'_{1,2}$ with a 6-GHz real-time digital scope. Therefore, the measured signals correspond to the filtered MZ output intensities, according to the combined filtering of $PD'_{1,2}$ and the scope. Additionally, the free input of the 2×2 couplers can be used to inject an optical signal (from laser $L'_{1,2}$), acting as an external perturbation.

For complementary insight into the behavior, we perform numerical modeling based on an integro-differential equation driven by a nonlinear delayed feedback term [18]. Describing the actually observed electrical signals $x_1(t)$ of PD'_1, the following model can be derived:

$$\tau\dot{x}_1(t) + x_1(t) + \frac{1}{\theta}\int_{t_0}^{t} x_1(s)ds = \beta_1 \cos^2[\alpha_1 x_1(t-T_1) + \alpha_c x_2(t-T_c) + \Phi_1], \qquad (1)$$

where τ and θ denote the characteristic times of the bandpass filter introduced by the photodiode, with $\tau=1/(2\pi f_h) \simeq 13$ ps and $\theta = 1/(2\pi f_l) \simeq 5.3$ μs corresponding to the high- and low-frequency cutoff. The delay T_1 is defined by the closed loop path from MZ_1, via the circulator C_1, the mirror, back to the circulator, to the photodiode PD_1, and the rf driver with gain G driving MZ_1. The delay T_c is the coupling time between the systems when the signal travels from MZ_2 via C_2, the mirror, C_1, PD_1, to the rf driver acting on MZ_1. The gain of the nonlinear transformation β_1 is practically tuned via the laser intensity seeding MZ_1. The scaling factors α_1 and α_c reflect the actual gain and loss asymmetry between the self-feedback and the cross coupling. For the presented results, these parameters have been accurately matched, and thus $\alpha_{1,2}=\alpha_c=1$. Finally, the phase Φ_1 is controlled by the bias of MZ_1; it determines the local shape of the nonlinear transformation.

The equation for the other observed signal at MZ_2 output is similar to Eq. (1) (interchanged indices 1 and 2). Equation (1) holds for identical photodiodes $P_{1,2}$ and $P'_{1,2}$ (same τ and θ), which is supported in practice by the use of the same photodiode model. Note that the matched pairs of rf drivers and MZ have substantially higher bandwidth, so that their influence on the dynamics can be adiabatically eliminated.

First, we investigate synchronization capabilities for almost symmetric conditions—i.e., for a shared feedback-coupling mirror almost in the middle of the coupling path: the coupling times are matched such that $T_{c1,c2}=T_c=(T_1+T_2)/2$. Additionally, the oscillator parameters are matched within a 2% accuracy, but a small self-feedback delay asymmetry is deliberately introduced, $T_1=108.95$ ns and $T_2=122.65$ ns. We adjust the parameters for broadband chaotic dynamics spanning the full bandwidth. This is achieved for a gain parameter $\beta_{1,2}=2.4$ and matched bias of both MZ at $\Phi_{1,2}=-0.25$ rad. For these conditions we measure the intensity dynamics and calculate the corresponding cross-correlation function $C_{12}(\Delta t)$ from 5-μs-long time series. It is defined such that maximal correlation C_{max} at $\Delta t_0 < 0$ implies lagging dynamics of system 2.

The experimentally obtained $C_{12}(\Delta t)$ and the intensity time series are presented in Figs. 2(a) and 2(b), respectively. $C_{12}(\Delta t)$ reveals characteristic peaks which can be assigned to the delays of the system. First, we identify a remarkably high $C_{max}=0.98$ at $\Delta t_0=(T_1-T_2)/2=-6.85$ ns. In the limit of matching delays, this offset vanishes leading to zero-lag synchronization. Second, the symmetric functions show recurring (anti)correlations at both feedback delays $T_{1,2}$ and multiples thereof, as well as (anti)correlations related to the difference of T_1 and T_2. The time series, depicted in Fig. 2(b), exhibits almost identical dynamics when compensated by Δt_0. This lag defines a condition for an intrinsic common drive signal, serving as a synchronization-stabilizing mechanism [15], because both systems are indeed driven by the same signals coming from the relay element (mirror), so that any dynamical output is equally distributed to both systems. Even for parameter detuning of several percent, the dynamics remains highly correlated with $C_{12} > 0.90$. This indicates stable identical synchronization. To substantiate this we numerically integrate the model equations using the experimental parameters $T_1=108.95$ ns, $T_2=122.65$ ns, $\beta_{1,2}=2.4$, $\Phi_{1,2}=-0.25$ rad, $\tau_{1,2}=13$ ps, and $\theta_{1,2}=5.3$ μs. Figures 2(c) and 2(d) show excellent agreement with the experiment. Indeed, for the matched parameters we obtain indistinguishable dynamics with $C_{max}=1.0$.

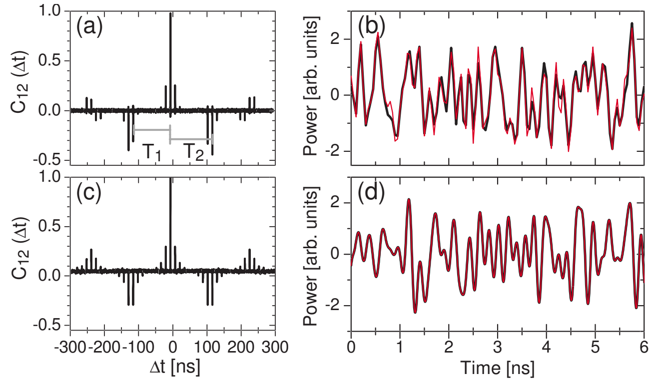

FIG. 2. (Color online) Cross-correlation functions from experiments (a) and numerics (c) and the corresponding time series in (b) and (d), $x_1(t)$ (black) and $x_2(t-\Delta t_0)$ (red).

Following the investigation of the almost-symmetric coupling condition, we discuss the influence of two key parameters. First, we explore whether stable identical synchronization is maintained for substantial delay mismatch, for which the solitary oscillators would exhibit different dynamics. Second, we explore how the synchronization properties change if we detune the nonlinear transformations of one oscillator with respect to the other. To answer the first question, we increase the delay for system 2 such that $T_2=1102.9$ ns exceeds $T_1=124.1$ ns by one order of magnitude. Again, we match the remaining parameters, maintaining $\beta_{1,2}=2.4$. To see whether the absolute phase of $MZ_{1,2}$ affects the synchronization quality, we choose a different but matched phase of $MZ_{1,2}$ corresponding to $\Phi_{1,2}=\pi/4$. For this condition, we measure the intensity time series and calculate $C_{12}(\Delta t)$, which is depicted in Fig. 3(a); the corresponding modeling result is shown in Fig. 3(b).

Both curves are in good agreement and illustrate highly correlated dynamics with $C_{max}=0.94$ (a) and 1.0 (b), despite the large delay mismatch. In particular, we see that the large

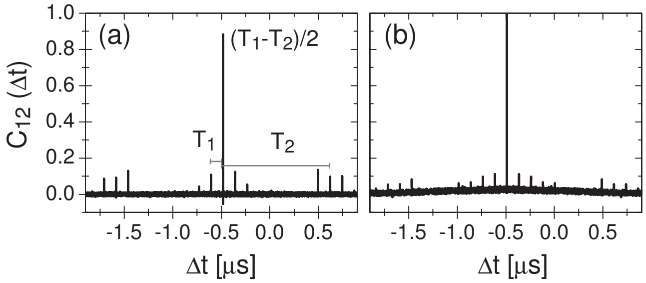

FIG. 3. Cross-correlation properties for strongly asymmetric delays $T_1 \ll T_2$: (a) experiment, (b) modeling.

delay mismatch results in a large relative temporal shift Δt_0. A comparison with Fig. 2 shows that the synchronization quality is not influenced by equally changing $\Phi_{1,2}$, while the overall correlation behavior can be affected due to a resulting change of the dynamics. These findings indicate stable identical synchronization for systems with strongly differing delays. Such less restrictive conditions might be important for synchronization in real-world delay systems and attractive for applications.

To gain insight into the stability properties of the synchronization, we study the system response to an external perturbation. This is realized via optical injection of an arbitrary bit-pattern signal of 500 Mbit/s (from L_1') into system 1. We choose a strong perturbation with an amplitude of 80% of the chaotic signal. For this condition, we characterize the temporally resolved synchronization quality. Therefore, we calculate the cross-correlation coefficient of 4-ns-long segments of the Δt_0-shifted time series. This is repeated for time steps of 0.05 ns unveiling the temporal evolution of C_{12}. The result is presented by the upper (red) curve in Fig. 4 which is depicted together with the perturbation (lower gray curve) and the difference signal of the detected dynamics. To facilitate comparison, only the difference signal, presented in black, has been low pass filtered to 1 GHz bandwidth. This signal resembles the original perturbation signal well. The cross-correlation values are mostly close to 1 and show low variance, indicating excellent synchronization. Only during fast perturbation changes, as occurring at the flanks of the bit pattern, do short correlation drops arise due to temporarily degraded synchronization. A statistical analysis of the mean duration of such degradation, which we define as being significant for $C_{12}(t) < 0.8$, yields a resynchronization time of

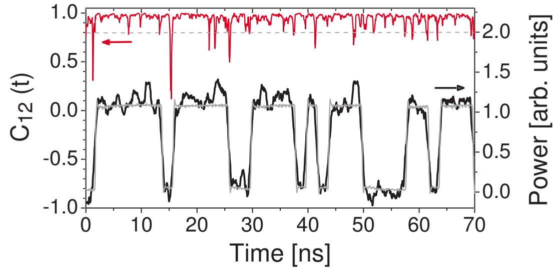

FIG. 4. (Color online) Perturbation signal (lower, gray), synchronization error $[x_1(t) - x_2(t - \Delta T_0)]$ (lower, black), and time-resolved cross correlation (upper, red line), with $T_1 \ll T_2$ as in Fig. 3.

$<$220 ps. This highlights an extremely fast resynchronization process if compared to $T_{1,2}$. We note that our result for almost-symmetric delays indicates even higher robustness.

The synchronization robustness can be attributed to the DSFC scheme, since the mirror acts as a symmetrically redistributing element; i.e., the feedback signal for one system represents the coupling signal for the other system and vice versa. Therefore, at any time, both oscillators are commonly driven by the same signals, only with a temporal offset given by $\Delta t_0 = (T_1 - T_2)/2$. This symmetric redistribution of the signals acts as a stabilizing (controlling) mechanism so that the dynamics of both oscillators adapt to the same collective dynamics, manifesting itself as synchronization. In the same way, asymmetric perturbations of the dynamics are also equally redistributed to both oscillators, efficiently reenforcing synchronization. Accordingly, we find robust identical synchronization. In contrast to this, we observe fundamentally different behavior when we remove the mirror and the 2×1 coupler, directly connecting the systems with a single fiber. Then, we only find generalized synchronization with the characteristic leader laggard dynamics, as is typically observed in mutually delay coupled systems [4,6]. For such coupling, the isochronous identical synchronization solution is unstable.

For our scheme, we emphasize two points. First, the condition for common drive can be generalized if we allow not only for dissimilar feedback delays $T_1 \neq T_2$, but also for dissimilar coupling times $T_{c1} \neq T_{c2}$. Deriving again the condition for common drive by simple substitution of time reference $[I_1(t-T_1) + I_2(t-T_{c1}) = I_2(t'-T_{c2}) + I_1(t'-T_2)]$, one can show analytically that the necessary condition for identical synchronization is $T_1 + T_2 = T_{c1} + T_{c2}$. The temporal offset is then given by $\Delta t_{0,g} = [(T_1 - T_2) - (T_{c1} - T_{c2})]/2$. We have verified this by inserting additional optical fiber between MZ_1 and C_1. Indeed, we obtain identical synchronization with a time offset given by $\Delta t_{0,g}$. Second, note that although this synchronization condition was derived as necessary, it is not a sufficient one [19].

Finally, we discuss the synchronization properties for mismatching the nonlinear transformations of the two oscillators. This is made possible through the independent tuning of the local shape of the $\cos^2$ nonlinearity; the related detuning parameter is then the relative phase $\Delta\Phi = \Phi_1 - \Phi_2$, which influences the feedback and coupling conditions. The symmetry and the π periodicity of the nonlinearity suggest characteristic phenomena for $\Delta\Phi = |\pi/4|$ and $\Delta\Phi = |\pi/2|$, which we discuss in the following. As a starting point we choose the condition for which we find identical synchronization for $\Delta\Phi = 0$, corresponding to Fig. 2. From this condition, we decrease Φ_2 until $\Delta\Phi = \pi/2$, for which we obtain good agreement between the experimental and numerical cross correlations [Figs. 5(a) and 5(b), respectively]. Both reveal anticorrelated dynamics with $C_{min} = -0.93$ (a) and -1.0 (b) at Δt_0, the same offset found C_{max} for identical synchronization. For this antisynchronization state we find similar fast resynchronization times as for the identical synchronization, due to the stabilizing common drive. An interesting intermediate state can be observed for $\Delta\Phi = \pm \pi/4$, for which the two nonlinear functions are in quadrature. Figures 5(c) and 5(d) present the experimental and numerical cross-

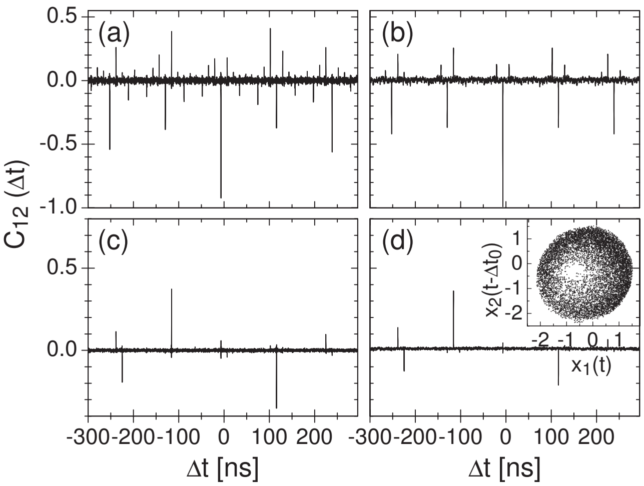

FIG. 5. Experimental cross correlation for $T_1 \approx T_2$, but $\Delta\Phi \neq 0$: (a) $\Delta\Phi = \pi/2$ and (c) $\Delta\Phi = \pi/4$. Corresponding modeling results are in (b) and (d). The inset in (d) is a circular synchronization plot.

correlation results ($\Delta\Phi = +\pi/4$). This particular detuning leads to vanishing linear cross correlation at Δt_0, but with remaining correlations related to $T_{1,2}$. Further insight into this interesting state can be obtained from the corresponding synchronization plot depicted as an inset in Fig. 5(d): the dynamics reveals a robust circular attractor structure. These results exclude unsynchronized dynamics which would manifest itself in filling the complete phase portrait: we conclude the presence of stable generalized synchronization. This particular synchronization state emerges from a crescent-shaped structure for increasing $\beta_{1,2}$, which closes to a circular one as soon as two maxima of the nonlinear functions are involved in the dynamics. For detuning from $\Delta\Phi = 0$ to $\Delta\Phi = \pi/2$, we find gradual transitions between the reported states: the synchronization plot first gives rise to linearly correlated dynamics, and then it takes up elliptic shape which continuously deforms to a circular one.

Altogether, the presented system is attractive, since it allows for complementary experimental and theoretical studies of the transitions between different synchronization states. Such fundamental insight is desired for the understanding of dynamical phenomena in various systems in nature [20], but it can also serve functional purposes such as in chaos-based communication.

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